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**BULLETIN 476**

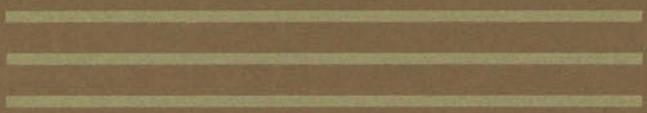
**RHEOLOGY OF CONCRETE:  
A REVIEW OF RESEARCH**

by

I. Ali

and

C. E. Kesler







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by

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conducted by  
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in cooperation with

The State of Illinois  
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The U.S. Department of Commerce  
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## ABSTRACT

CONCRETE IS A MATERIAL EXHIBITING TIME DEPENDENT STRESS-STRAIN CHARACTERISTICS WHICH ARE OF INTEREST TO ENGINEERS AND DESIGNERS USING CONCRETE AS A MATERIAL OF CONSTRUCTION. THE TIME-DEPENDENT STRAIN INDUCED BY AN APPLIED SUSTAINED STRESS, COMMONLY TERMED CREEP, AND THE VARIOUS FACTORS AFFECTING IT ARE DISCUSSED. A COMPREHENSIVE REVIEW OF OUR PRESENT KNOWLEDGE OF THE RHEOLOGICAL BEHAVIOR OF CONCRETE AND THE VARIOUS HYPOTHESES ADVANCED BY NUMEROUS INVESTIGATORS ARE PRESENTED. THE BIBLIOGRAPHY CONTAINS OVER 207 REFERENCES DATING BACK TO THE FIRST DECADE OF THIS CENTURY. THIS REVIEW OF PAST INVESTIGATIONS INTO THE RHEOLOGICAL BEHAVIOR OF CONCRETE INDICATES THAT A WIDE VARIETY OF APPROACHES HAVE BEEN USED RESULTING IN THE ACCUMULATION OF LARGE MASSES OF DATA. ALTHOUGH MUCH KNOWLEDGE HAS BEEN GAINED, CONSIDERABLE DIFFERENCES OF OPINION STILL EXIST REGARDING MANY ASPECTS OF CREEP PHENOMENA. AN ATTEMPT IS MADE TO CONSOLIDATE OPINIONS ON FACTORS AFFECTING CREEP, THE RELATION OF CREEP TO STRUCTURAL BEHAVIOR, THE ORIGINS OF CREEP, AND THE RHEOLOGICAL MODELS FOR CREEP WHICH HAVE BEEN PRESENTED BY VARIOUS INVESTIGATORS. SOME PROMISING AREAS FOR FURTHER WORK ARE ALSO PRESENTED.

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On behalf of the University, the work covered by this report was carried out under the general administrative supervision of W. L. Everitt, Dean of the College of Engineering, R. J. Martin, Director of the Engineering Experiment Station, T. J. Dolan, Head of the Department of Theoretical and Applied Mechanics, and Ellis Danner, Director of the Illinois Cooperative Highway Research Program and Professor of Highway Engineering.

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## I. INTRODUCTION

### A. PURPOSE AND SCOPE OF THE REVIEW

The strain exhibited by a material in response to a given applied stress may, within the limits of observation, either be constant with respect to time or vary with it. Most engineering materials fall in the second category, especially in certain environments like elevated temperatures or high stresses.

Concrete exhibits an unquestionable influence of time on its stress-strain relation. In view of the extensive use of concrete as a structural material, the importance and necessity of studying its rheological behavior are obvious. The engineer may be concerned either with the time-dependent strains induced by a given applied stress or the time-dependent stresses mobilized under a given applied strain. The former is commonly termed the creep problem and the latter the relaxation problem. The two problems are essentially interrelated and constitute two aspects of the general rheological behavior of concrete.

As mentioned above, creep and relaxation are interrelated phenomena. Under certain circumstances it may not even be possible to define a problem solely in terms of one or the other. For reasons of convenience, however, the present review will deal primarily with the phenomenon of creep, defined as the stress-time-dependent deformation of concrete, especially under constant non-zero stress.

This report is an attempt to review our

present knowledge about the phenomenon of creep and its practical implications, to discuss and evaluate the various hypotheses advanced to explain its nature, and to delineate, wherever possible, areas in which opinions differ and directions in which further clarification might be desirable.

Special emphasis has been given to the representation of concrete as a viscoelastic material. The possibility of correlating the internal damping and the creep behavior of concrete, and some recent investigations in this direction, have also been discussed.

### B. HISTORICAL BACKGROUND

Woolson <sup>(1)\*</sup> was perhaps the first to describe, in 1905, tests on the "flow of concrete under pressure."

Hatt, <sup>(2)</sup> in 1907, reported results of tests on 8-by 10-in. reinforced concrete beams loaded at third points at the age of about 60 days. He found the two-month deflections about twice the initial deflections.

McMillan, 1913-15, <sup>(3)</sup> reported tests on 30 in. by 5½-in. by 12-ft. reinforced concrete beams. Like Hatt, he found 60-day midspan deflection to be twice the initial deflection. Six hundred-day deformation (excluding shrinkage measured from transverse measurements) was found to be three times the instantaneous deformation due to the dead and live loads,

\*Parenthetical numbers refer to the entries in the References.

the live load having been sustained from 107 days on. Occasional unloading gave a measure of instantaneous elastic deformation.

In 1915-16, Fuller and More<sup>(4)</sup> attempted to simulate, by means of laboratory tests, time-dependent deformations observed in a reinforced concrete building. Results were complicated by the fact that the influencing conditions were probably not effectively reproduced and some tests were performed on specimens in which residual flow from previous tests was still effective.

Smith,<sup>(5)</sup> in 1914-16, confirmed findings of other authors' creep experiments and noted that plastic recovery was small as compared with the initial creep.

McMillan,<sup>(6)</sup> in 1916, reported tests on beams and plates, and observed that up to two years and more, creep was still taking place and that the deflections were already 3 to 4 times the instantaneous deflections.

Goldbeck and Smith,<sup>(7)</sup> in 1916, published the results of their tests. They found that creep under water was smaller than in air, and that a part of creep was recoverable. They noted the recovery to be slow and not complete. They found six-month deformation to be three times, and two-year deformations to be four times, the corresponding instantaneous deformations.

Smith,<sup>(8)</sup> in 1917, noted stress relaxation in concrete. He established that creep shows an asymptotic trend. He observed the total deformation of gravel concrete to be 20 to 30 per cent greater than that of limestone concrete. McMillan, in his discussion of the paper by Smith, reported high steel stresses resulting from creep of concrete in columns.

Lord,<sup>(9)</sup> in 1917, reported results of tests made on a 25-ft. by 26-ft. slab, 10 in. thick. He found 55-day deformations to be  $2\frac{1}{2}$  times those at one day and 230-day deformations

to be five times the latter.

Hollister,<sup>(10)</sup> in 1919, reported tests on a reinforced concrete slab and observed that the plastic deformations, defined as residual deformations after unloading, were, for all loads, approximately equal to the corresponding elastic deformations.

McMillan,<sup>(11)</sup> in 1921, reviewed his findings on creep, along with the findings of other investigators. He pointed out that as concrete crept, the load carried by the reinforcing steel increased. In the discussion of McMillan's paper, Lagaard reported data showing continued shortening of concrete columns even after six years. Steel stresses in the vertical steel increased by up to 1500 psi due to creep and shrinkage. Lateral shrinkage effects were also of the same magnitude. The phenomenon was subsequently observed by many other investigators.

Clemmer,<sup>(12)</sup> 1923, reported tests in which the effect of sustained load on ultimate strength was noted. At 71 per cent of the ultimate, the cantilever beams he tested failed after 105 hours. At 84 per cent of ultimate it took only 14 hours. He noted that much of the creep deformation was recoverable eventually.

Faber,<sup>(13)</sup> in 1927, reported tests on six 2-in. by 5-in. by 15-ft. long beams. Large deflections were observed after six months. Effect of shrinkage was noted, which caused the top fibers to be considerably compressed and the bottom fibers to be relieved to some extent. He thus separated shrinkage and creep and suggested that shrinkage was independent of stress.

Freyssinet<sup>(14-19)</sup> carried out several investigations to study creep and published, during 1930-36, a series of papers on the subject of deformation in concrete.

The extensive work of Davis and Davis<sup>(20)</sup>

in the United States, and Glanville<sup>(21)</sup> in Britain, during the period 1925-31, was especially significant. Herein the variables were systematically studied and much reliable information was obtained.

Davis and Davis<sup>(20)</sup> in their 1931 paper reported tests on a large number of specimens under sustained loading. They included most of the major variables in the tests and presented conclusions about their effect on creep. Unloaded "control" specimens were provided to account for volume changes other than those due to load. They noted that short-period recovery equaled short-period creep more closely as the age of concrete increased.

Glanville,<sup>(21)</sup> in 1930, reported tests on a large number of plain and reinforced concrete specimens in controlled environments. He studied most of the important variables as Davis and Davis did. The definition of creep that he adopted was different from that adopted by most American investigators. He deducted shrinkage and the instantaneous elastic deformation at the age of measurement of creep. For use in design calculations, he introduced the concept of the "effective" modulus of elasticity, though he recognized its limitations.

A large number of papers have been written on the nature of creep, on mathematical expressions for creep deformation, on effect of creep in reinforced concrete structures, and on numerous other aspects. Only a few of these will be mentioned below.

Straub,<sup>(22)</sup> in 1931, presented a mathematical expression relating creep strain, applied stress, and elapsed time, which implied that creep strains were not necessarily in linear proportion to the applied stress. He also pointed out the possible stress relief due to creep in indeterminate reinforced concrete structures.

Bingham and Reiner,<sup>(23)</sup> in 1933, conducted tests on cement and cement-mortar beams and studied creep in concrete as a rheological problem. They postulated that the behavior of hardened cement paste was analogous to that of a viscous liquid.

Thomas,<sup>(24)</sup> in 1933, presented an exponential expression for creep, indicating that creep of concrete did approach a limiting value.

Davis, Davis and Hamilton,<sup>(25)</sup> in 1934, published results of tests on concrete cylinders and columns under loads sustained for seven years. Their tests included most of the major variables that affect creep in concrete. They also studied the increase of stress in steel in reinforced concrete columns. They discussed the nature of creep and shrinkage and conditions under which creep may be important in structural design.

Lynam,<sup>(26)</sup> in 1934, in his book, "Growth and Movement in Portland Cement Concrete," proposed an organic concept of creep. He explained the phenomenon of creep from a colloidal point of view and stated that the squeezing out of water from gel under pressure, very much as in silica gel, was responsible for creep in concrete and that creep and shrinkage were two aspects of essentially the same phenomenon. He maintained that creep could not be explained on the basis of viscous flow or crystalline slip.

Richart and Brown,<sup>(27)</sup> in 1934, showed that in reinforced concrete columns shrinkage and creep increased stresses in reinforcing steel but had no effect on the ultimate strength of the columns.

Busch,<sup>(28)</sup> in 1937, published a paper treating the effects of concrete composition on creep and creep behavior of reinforced concrete.

Shank<sup>(29, 30)</sup> carried out extensive work

on the creep of concrete. He analyzed the then available creep data and evolved a creep formula, expressing creep as proportional to stress and a fractional power of time. He presented experimental values for coefficients in the formula under different conditions and explained application of the formula to the design of reinforced concrete members.

Ross,<sup>(31)</sup> in 1937, carried out extensive work on the subject of creep of concrete and developed a hyperbolic expression for expressing the magnitude of creep strain as a function of time. He suggested values of constants for use under a variety of conditions.

Theuer,<sup>(32)</sup> in 1937, carried out experiments to determine the effect of temperature on creep of concrete. He found that temperature had no appreciable effect on dry specimens, while wet and half-dry specimens showed an increase in creep with increase of temperature.

Davis, Davis and Brown,<sup>(33)</sup> in 1937, reported results of their long-time loading tests on concrete specimens. Findings on the influence of density of cement paste, creep in tension, and influence of type of cement were described.

Jensen and Richart,<sup>(34)</sup> in 1938, studied creep in compression of concrete for very short periods of time compared with available long-time test results. They found that there was a continuity between the short-time and long-time creep and they were essentially similar in nature. Specific creep, i.e., creep per unit stress, was found to be in approximately inverse proportion to the strength. They found the creep-time curve to approximate a second degree parabola.

Glanville and Thomas,<sup>(35)</sup> in 1939, reported their findings on creep in tension, lateral deformation under compression, and effect of creep on the deformation and ultimate

strength of reinforced concrete beams. They also reported vertical and horizontal deformations of a reinforced concrete arch of a large hall in London.

Lorman,<sup>(36)</sup> in 1940, developed a hyperbolic relation defining the creep-time relation for concrete. Correlating available creep data with the constants of the hyperbolic formula, he tried to establish fundamental principles concerning the manner in which creep constants are modified by various physical factors. He presented two methods of calculating the creep constants, one for constant loads and the other for variable loads.

Maney,<sup>(37)</sup> in 1941, stated that the so-called inelastic deformation due to load was actually an additional elastic deformation due to non-uniform shrinkage and that the confusion lay in the incorrect interpretation of the differential time-dependent deformations of the loaded and unloaded specimens.

Pickett,<sup>(38)</sup> in 1942, presented a mathematical analysis in which the interaction between creep and shrinkage was shown to be a consequence of nonuniform shrinkage and a non-linear stress-strain relationship.

McHenry,<sup>(39)</sup> in 1943, studied the problem of creep and creep recovery and put forward his well-known principle of superposition of creep strains in concrete. He presented an exponential expression for creep and developed a method for solving certain structural design problems requiring a knowledge of creep strains.

Ross,<sup>(40)</sup> in 1943, presented an idealized mechanical model simulating the creep behavior of concrete. He developed equations based on this concept for evaluating shrinkage and creep in plain and reinforced concrete.

Duke and Davis,<sup>(41)</sup> in 1944, studied creep behavior of concrete under combined



stresses and torsion. They presented a method for estimating the magnitude of creep strains under combined stresses from the results of tests under uniaxial compression.

Arnstein and Reiner,<sup>(42)</sup> in 1945, reported their findings on creep in cement and cement mortar. They considered hardened cement paste, mortar, and concrete as very viscous liquids, the viscosity of which might be calculated from the dimensions of the specimen and the observed creep.

L'Hermite and LeCamus,<sup>(43, 44)</sup> in 1945, carried out important investigations on creep in concrete and reinforced concrete.

Shank,<sup>(45)</sup> in 1945, showed that creep strains increased very rapidly to very high values at or near the ultimate strength of concrete.

Ross,<sup>(46)</sup> in 1946, reported his findings on the effect of creep on strength of columns. He made studies of plastic models for this and found that the ultimate carrying capacity of a column was a function of time.

Washa,<sup>(47)</sup> in 1947, reported his findings on thin reinforced concrete slabs loaded for five years and, referring to the large deflections and strains observed, he emphasized the importance of the creep problem in thin slabs.

Seed,<sup>(48)</sup> in 1948, studied four different ways of estimating creep deformations in reinforced concrete structures under variable loads, namely the effective modulus method, Glanville's rate of creep method, Lorman's method of superposition, and Ross' method of using a mechanical model.

Reiner,<sup>(49)</sup> in 1949, in his book, "Deformation and Flow," devoted a chapter to the study of creep properties of cement and concrete. He considered hardened cement paste to be essentially a liquid of high viscosity, and applied Einstein's equation to derive the viscosity of cement mortar in terms of the

aggregate content and viscosity of hardened cement paste. He also considered creep to be due to isotropic or volume flow.

Blanks and McHenry,<sup>(50)</sup> in 1949, tested specimens of concrete with various types of openings and found that stress concentration was relieved by the creep of concrete, especially in the case of concrete loaded at an early age.

Dischinger,<sup>(51)</sup> in 1949, presented an exponential creep-time expression and studied the influence of creep on large-span arches and beams.

Vogt,<sup>(52)</sup> in 1950, as did Pickett in 1942, pointed out the interdependence of shrinkage and creep deformation as a result of nonuniform shrinkage stresses and nonlinear stress-strain relations.

Friedrich,<sup>(53)</sup> in 1950, proposed a power expression for the creep-time relation.

Washa and Fluck,<sup>(54)</sup> in 1950, investigated the change in the elastic and strength properties of concrete specimens under sustained loads. They found modulus of elasticity to be increased after 10½ years of sustained loading. Compressive strength for vibrated concrete was found to be five per cent lower, while that for hand-rodded concrete was five per cent higher after the said period of sustained loading. They found creep to be greater in magnitude for hand-rodded concrete.

Flügge,<sup>(55)</sup> in 1950, analyzed creep data from tests by Glanville and Davis and Davis and derived coefficients for the elements of a mechanical model that he suggested for representing the creep behavior of concrete.

Freudenthal,<sup>(56)</sup> in 1950, described concrete as a viscoelastic material and attributed the properties of creep and relaxation in concrete to its viscoelastic nature.

Freyssinet,<sup>(57)</sup> in 1951, presented a somewhat different concept of shrinkage and

creep. He considered concrete as a complex "solid-liquid" substance to which the laws of thermodynamics may be applied. He explained the phenomena of shrinkage, creep, elastic deformation, and autogenous healing on the basis of his theory.

Lee,<sup>(58)</sup> in 1951, suggested that creep in concrete can be regarded as a long-continuing viscous flow combined with a short-term delayed elastic effect.

Rao,<sup>(59)</sup> in 1951, postulated that creep was mostly due to shrinkage caused by loss of moisture, which is considerably reduced when the specimen is coated with asphalt or kept wet. He attributed the additional time-dependent deformation in a reinforced concrete beam to the shrinkage that is restrained by the reinforcement.

Ross,<sup>(60)</sup> in 1951, described tests designed to produce, by initial dessication, an expanding concrete which, if it could be produced on a practical scale, would eliminate the problem of loss of prestress in steel.

Washa and Fluck,<sup>(61)</sup> in 1952, demonstrated the beneficial effect, regarding strains and deflections, of adding arbitrary amounts of compressive reinforcement in reinforced concrete beams, and suggested use of compressive reinforcement when large span-to-depth ratios and sustained loads are involved.

Aroutiounian,<sup>(62)</sup> in 1952, discussed the problem of creep in concrete and gave exponential functions to define the creep-time relation. He assumed the principle of superposition and linearity of creep strain with stress up to half of the ultimate strength.

Kubo,<sup>(63)</sup> in 1952, attempted to investigate the fundamental equation by which plastic deformation and creep of cement mortar occurring during a short period could be solved. He derived an equation connecting stress, strain, and time and experimentally verified it

by tests on cement mortar.

Koerner,<sup>(64)</sup> in 1953, studied the influence of time on the strain of rigid materials. He applied Maxwell's relaxation theory to the problem and compared the results with creep-time relations given by other authors, and concluded that Maxwell's theory may be applied not only to concrete but to other materials as well.

United States Bureau of Reclamation,<sup>(65)</sup> in 1953, reported their findings on the creep of mass concrete, on the basis of tests carried out over a period of 10 years. Significant relationship was observed between creep strains and corresponding elastic strains. Little or no correlation could be obtained between the strength of concrete and creep.

Torroja and Paez,<sup>(66)</sup> in 1954, wrote an excellent chapter on the rheology of set concrete. They described the structure of set cement paste and the state of water in it, and discussed the problem of volume change on this basis. They reviewed explanations given by various authors for the causes of volume changes. They devised a complex rheological model to explain the behavior of concrete under load, and obtained an encouraging degree of correspondence between theoretical and experimental values.

Ross,<sup>(67)</sup> in 1954, studied the problem of creep under two-dimensional loading. He found that under sustained two-dimensional stress, creep strains are induced in the principal directions having magnitudes of approximately the same order as the corresponding creep under uniaxial stress; Poisson's ratio for creep is almost zero.

McCoy,<sup>(68)</sup> in 1955, reviewed, on behalf of the Waterways Experiment Station, Mississippi, the literature on creep in concrete. A logarithmic relation based on the assumption that creep rate is inversely proportional to

time was found to be simpler and fitted available experimental data more closely than an exponential equation based on the assumption of the creep rate being proportional to the amount of creep remaining at that time.

Neville,<sup>(69)</sup> in 1955, summarized various theories of creep such as seepage, viscous flow, crystalline flow, etc. He concluded that what is normally measured as creep consists partly of true creep, which is viscous in nature, and partly of increased shrinkage due to evaporation and external load; also that the magnitude of creep depended on the applied stress and the properties of concrete but was independent of other factors such as curing conditions, humidity, temperature, etc. which affected only the shrinkage.

Chang and Kesler,<sup>(70, 71)</sup> in 1956, published papers wherein they presented nonlinear mechanical models to represent creep and relaxation behavior of concrete. Coefficients of the models were correlated with dynamic properties of concrete.

Erzen,<sup>(72)</sup> in 1956, presented an expression describing the total strain in concrete and used it for calculating the loss of stress in prestressed beams. He showed that loss of prestress could be calculated if variation of the modulus of elasticity with time and the creep equation of concrete were known.

Freudenthal,<sup>(73)</sup> in 1956, discussed effects of creep in the analysis of reinforced concrete structures. He showed that creep sensitivity of reinforced concrete structures was a function of two factors: the ratio of sustained stress to total stress, which he termed the structural component of the creep sensitivity, and the ratio of the total stress to the strength of concrete, which he termed the material component. He presented a rheological model containing eight elements and derived values of their coefficients for dif-

ferent conditions.

L'Hermite,<sup>(74)</sup> in 1957, reviewed the knowledge of creep of concrete with special reference to tests carried out by him, LeCamus, Griangreco, and others at the Technical Institute of Buildings and Public Works in Paris. He discussed creep under sustained compression, tension, repeated loads, flexure, and combined stresses. He reviewed available theories of creep in concrete and offered his own explanation of the phenomenon. He also presented an exponential form for the creep-time relation, based on the assumption that viscosity of concrete is partly constant and partly an inverse function of time.

Ross,<sup>(75)</sup> in 1958, presented three methods for computing creep under variable stress from results of tests under constant stress. He showed that each method had certain advantages in particular circumstances depending upon the character of the stress variation, extent of available creep data, and the accuracy desired.

Troxell, Raphael, and Davis,<sup>(76)</sup> in 1958, reported their findings based on 30 years of creep study. Almost all major variables were included in their tests. Among other things they found that in some reinforced concrete columns under sustained loads, stress was transferred to steel to such an extent that the concrete was actually in tension.

Wagner,<sup>(77)</sup> in 1958, compiled, organized, and analyzed a large amount of data on creep and described in detail influences of various factors on the creep of plain concrete. He studied the influence of these factors separately on two creep parameters which he termed "creep-measure" and "creep-number," signifying respectively the absolute magnitude and the magnitude in relation to instantaneous deformation. He summarized the mathematical expressions put forward by various authors. He

analyzed 186 creep tests from different investigators and classified them according to the major variables studied, and expressed their results in terms of the creep-measure and creep-number.

Hansen,<sup>(78)</sup> in 1958, discussed some fundamental problems of creep. He pointed out the objection to measurement of creep on drying concrete under tensile, flexural, and high compressive loads. He emphasized the severe influence of moisture diffusion on creep and introduced the concept of what he termed "basic creep," signifying creep when no moisture diffusion takes place. He considered concrete to be viscoelastic and presented a formula relating the modulus of elasticity of concrete to that of its components. In a later paper<sup>(79)</sup> in 1960 he developed a relationship between creep and the gel density and the volumetric concentration of cement paste in concrete, and proposed a rheological model to simulate the creep behavior of concrete.

Bibliographic information is available in compilations by Slate,<sup>(81)</sup> Corley,<sup>(82)</sup> Meyers,<sup>(83)</sup> and an unpublished report of the Portland Cement Association.<sup>(84)</sup> Publications by L'Hermite,<sup>(74)</sup> Wagner,<sup>(77)</sup> and Fluck and Washa<sup>(85)</sup> are supplemented with excellent lists of references. The publications by

Torroja and Paez,<sup>(66)</sup> Hansen,<sup>(78, 79, 80)</sup> Lea and Lee,<sup>(86)</sup> and Neville,<sup>(69)</sup> among others, deserve special mention for presenting a general coordinated view of the problem of creep in concrete.

Many investigators have attempted to tackle various aspects of the problem of creep in concrete. Much valuable information has been accumulated and several very significant hypotheses have been developed. The multiplicity of the factors involved and the complexity of creep and its associated problems have, however, eluded a full understanding of the phenomenon, and kept the field wide open for further research.

#### C. NOTATION

A variety of notations was encountered in the references included in this review. In order to avoid possible confusion, an attempt has been made to adopt, as far as practicable, a uniform notation for commonly used terms. Less frequently used terms could not be covered, and repetitive use of symbols for empirical constants and functions was unavoidable. All notation is defined in the text when first introduced and whenever necessary thereafter. The notation used is listed below:

$A$	= amplitude of free vibration, also area
$A_c$	= area of concrete
$A_s$	= area of steel
$A_n$	= amplitude of free vibration after $n$ cycles
$B_u$	= Burghers body
$C$	= creep or compliance operator such that $\epsilon_T = C'(\sigma_T)$ , also capacitance
$C_1, C_2,$ $C_3, C_4$	= canonic forms of rheological models
$c$	= specific creep, i.e., creep strain under unit stress, also damping coefficient, also subscript to denote creep or concrete
$c_\infty$	= ultimate value of specific creep
$c_c$	= critical damping coefficient
$c_K$	= damping coefficient of the Kelvin dashpot

$c_M$	= damping coefficient of the Maxwell dashpot
$E$	= elastic modulus
$E_c$	= elastic modulus of concrete
$E_d$	= dynamic elastic modulus
$E_h$	= elastic modulus of the hard phase in a composite body
$E_s$	= elastic modulus of steel, also of the soft phase in a composite body
$E_2$	= elastic modulus of aggregate
$E_4$	= elastic modulus of unhydrated cement particles
$E_5$	= elastic modulus of cement gel
$e$	= base of natural logarithms, also electromotive force, also subscript to denote elastic
$F$	= force
$F_o$	= initial force
$F_t$	= force after time, $t$
$f$	= frequency, also strength
$f_n$	= natural frequency of transverse vibration
$f'_n$	= natural frequency of longitudinal vibration
$f''_n$	= natural frequency of torsional vibration
$f_o$	= resonant frequency
$f'_c$	= compressive strength of concrete
$G$	= shear modulus
$G_d$	= dynamic shear modulus
$H$	= Hookean body
$I$	= moment of inertia of prismatic member cross section
$i$	= electrical current
$K$	= Kelvin body, also subscript for same
$k$	= spring stiffness, also age of concrete at loading
$k_K$	= spring stiffness of Kelvin spring
$k_M$	= spring stiffness of Maxwell spring
$L$	= inductance
$M$	= Maxwell body, also subscript for the same
$m$	= mass, also mass per unit length of prismatic member
$N$	= Newtonian body, also operator such that $\epsilon_x = C(\sigma_x) - N(\sigma_y + \sigma_z)$
$o$	= subscript for initial value
$P$	= total load
$p$	= force, also intensity of transverse load
$q$	= electrical charge
$R$	= the relaxation operator such that $\sigma_T = R(\epsilon_T)$ , also electrical resistance
$r$	= ratio of forcing frequency to natural frequency of a system
$S$	= total shrinkage force in concrete i.e., $s \cdot A_c$
$s$	= equivalent shrinkage stress
$T$	= any given time, also corresponding subscript
$t$	= elapsed time, also corresponding subscript

$V$	= volume
$V_a$	= volume of original air voids
$V_c$	= solid volume of cement content
$V_{hc}$	= gross volume of gel corresponding to full hydration of cement
$V_m$	= constant proportional to surface area of gel
$V_o$	= volume of original water content
$v$	= velocity, also volume concentration
$v_h$	= volume concentration of the hard phase in a composite body
$v_s$	= volume concentration of the soft phase in a composite body
$v_1$	= volume concentration of cement paste in the paste-aggregate system
$v_2$	= volume concentration of aggregate paste in the paste-aggregate system
$v_3$	= volume concentration of gel and capillary pores in the paste
$v_4$	= volume concentration of unhydrated cement in the paste
$v_5$	= volume concentration of gel in the gel and capillary pore system
$v_6$	= volume concentration of capillary pores in the gel and capillary pore system
$W$	= total energy of vibration
$w$	= deflection, also weight
$w_o$	= weight of original water content in concrete
$w_n$	= weight of non-evaporable water content in concrete
$X$	= amplitude of forced oscillations, also gel-space ratio
$X'$	= modified gel-space ratio
$x$	= displacement, also subscript for corresponding coordinate direction
$\alpha$	= spring compliance in rheological models, also degree of hydration of cement
$\alpha_e$	= instantaneous elastic compliance
$\alpha_c$	= ultimate creep compliance
$\alpha_s$	= compliance of reinforcing steel
$\alpha'_e, \alpha'_c, \alpha'_s$	= $\frac{\alpha_e}{A_c}, \frac{\alpha_c}{A_c}, \frac{\alpha_s}{A_s}$ respectively
$\alpha_K$	= compliance of Kelvin spring
$\alpha_M$	= compliance of Maxwell spring
$\gamma$	= shear strain, also with coordinate subscripts $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$
$\Delta$	= deformation, also small change
$\delta$	= logarithmic decrement, also $\frac{v_{nc}}{v_m}$
$\epsilon$	= strain, also with coordinate subscripts, i.e., $\epsilon_x, \epsilon_y, \epsilon_z$
$\epsilon_c$	= creep strain
$\epsilon_e$	= instantaneous elastic strain
$\epsilon_r$	= recovery strain
$\epsilon_u$	= strain at failure
$\epsilon'$	= fictitious strain in the elastic viscoelastic analogy
$\zeta$	= damping factor, i.e., $\frac{c}{c_c}$

$\eta$	= viscosity
$\theta$	= yield limit for St. Venant body
$\mu$	= Poisson's ratio
$\rho$	= density
$\sigma$	= normal stress, also with coordinate subscripts, i.e., $\sigma_x, \sigma_y, \sigma_z$
$\sigma_u$	= normal stress at failure
$\sigma'$	= fictitious stress in the elastic-viscoelastic analogy
$\tau$	= shear stress, also with coordinate subscripts, i.e., $\tau_{xy}, \tau_{xz}, \tau_{yz}$ , also retardation or relaxation time
$\tau_K$	= retardation time of Kelvin body
$\tau_M$	= relaxation time of Maxwell body
$\Phi$	= fluidity of dashpot, i.e., $\frac{1}{c}$
$\phi$	= phase angle
$\psi$	= specific damping capacity
$\omega$	= angular frequency
$\omega_n$	= undamped natural angular frequency

## II. FACTORS AFFECTING CREEP

### A. GENERAL

A large number of variables directly or indirectly affect the magnitude of creep. Extensive investigations, some of which have already been mentioned, have been carried out to determine the role different factors play in relation to creep of concrete. However, since the number of variables is large, and many of them are more or less covariant and interdependent, it is only for some variables that the opinions of various experimenters agree. In addition, most of the conclusions presented have been qualitative rather than quantitative, which may be attributed to factors such as inherent difficulties in quality control and the coexistence of shrinkage and elastic deformation with creep. Thus, whatever is stated concerning the effects of variables can be taken only as suggesting statistical trends rather than a statement of strict causal relations capable of general extension. This may, of course, be considered a question of degree, but the difference is almost like one of type in case of concrete, which is, as Lynam<sup>(26)</sup> put it, a material which exhibits properties of growth and movement almost like a living thing.

### B. PRINCIPAL VARIABLES

The principal variables affecting creep of concrete can be classified as follows:

#### A. Ingredients

##### 1. Cement

Composition

Fineness

##### 2. Aggregate

Permeability and absorption

Mineralogical composition

Particle size

Grading

Unit weight

Rheological properties

##### 3. Admixtures

### B. Mix proportions

#### 1. Cement paste content

#### 2. Water cement ratio

#### 3. Air content

### C. Mixing and Compaction

#### 1. Mixing time

#### 2. Method and extent of compaction

### D. Curing history until the time of loading

#### 1. Duration

#### 2. Temperature

#### 3. Humidity

### E. Stress

#### 1. Type

Compressive

Tensile

Flexural

Torsional

Multiaxial

#### 2. Magnitude

#### 3. Time-variation

#### 4. Duration



- F. Strength of concrete
- G. Hygrothermal conditions within the concrete
- H. Hygrothermodynamics or moisture movement in the concrete
- I. Temperature and humidity of the environment
- J. Shrinkage
- K. Shape and size of specimen
- L. Presence of reinforcement

#### C. EFFECTS OF VARIABLES

As the isolated effect of each variable is difficult to assess, the discussion of variables will be made in general terms, while broadly following the variables listed above in sequence.

##### 1. Ingredients

Concrete ingredients, viz., cement, water, aggregate, and air, all play a part in affecting creep to varying degrees.

a. Cement and Water: Cement composition, and fineness, have considerable influence on creep properties.

Drying shrinkage is dependent on composition more than on fineness, and the same may be expected of creep. Shrinkage is low for those cements which are high or moderately high in tricalcium silicate and for those low in alkalis. The amount of gypsum affects shrinkage appreciably. The higher the percentage of  $C_3A$ , the higher the optimum percentage of gypsum for minimum shrinkage, high  $C_3A$  or alkali cement being more sensitive to it. From the close resemblance between shrinkage and creep, and in the absence of comprehensive data, according to Davis and Troxell, <sup>(87)</sup> the same may be taken to be true of creep.

Davis, Davis, and Brown <sup>(33)</sup> studied the effect of fineness and composition of cement upon creep. They compared creep in normal portland cement and low-heat cement, and found that for a specific surface of 1800  $cm^2/gm$ ,

low heat portland cement exhibited, on the average, 80 per cent greater creep than normal portland cement. In tests on cements with specific surfaces of 1300 and 2200,  $cm^2/gm$ , it was found that coarsely ground low-heat cement exhibited greater creep and greater rate of creep at all ages. In case of low heat portland cement, creep was greater for the coarser cement while the reverse was true in case of normal portland cement, probably due to variables in particle sizes not manifest in the calculated values of the specific surfaces. Figure 1 shows the results reported by Troxell, Raphael, and Davis. <sup>(76)</sup>

Neville, <sup>(88)</sup> in his study of effect of cement properties on the creep of mortar, found that fineness of cement, per se, had no effect on creep properties, although increased fineness on regrinding caused an increase in creep indirectly due to reduction in gypsum content below the optimum. Increased fineness would be expected to accelerate hardening and cause a change in creep, but he could not verify this in his experiments.

Glanville, <sup>(21)</sup> studied three types of cement; namely, normal portland cement, rapid-hardening portland cement, and an aluminous cement. He found that the rapidity of hardening directly influences the creep to a considerable degree. The greater the rapidity of hardening, the lower the creep for a given age, provided the concrete is stored in air. Differences were not pronounced for storage under water.

Ross <sup>(89)</sup> found creep in cement made from blast furnace slag to be higher than that in normal portland cement.

Low-heat cement concrete has been found to creep more than normal cement at all ages. Washa <sup>(90)</sup> reports that low-heat cement reached a limiting value at five years as against two years in case of normal and high-early-strength

concretes and that addition of pozzolanic admixtures tended to increase creep.

Sheikin and Baskakov<sup>(91)</sup> studied the effect of cement composition on creep of concrete and found that specific creep\* varied widely, but for the same stress-strength ratio the creep was about the same. Lowest creep was recorded with cement consisting largely of alite\*\*, and the highest with cement consisting largely of belite\*\*, cement consisting largely of celite\*\* occupied an intermediate position. They found mineralogical composition to have ceased to influence creep after seven days.

Quantity of cement paste has a marked influence on creep. Creep increases directly as the quantity of cement paste. Troxell, Raphael, and Davis,<sup>(76)</sup> report that the percentage difference in creep of two mixes having the same water-cement ratio was about equal to the percentage difference in the paste content of the mix, Figure 2.

Glanville<sup>(21)</sup> studied the effect of cement content on creep characteristics using three mixes: 1:1:2, 1:2:4, and 1:3:6 by weight. He found (Figure 3) that creep was proportional to the quantity of aggregate used. This finding appears contrary to the observation by Davis mentioned above. Presumably, while changing the mix, Glanville kept the consistency constant and not the water-cement ratio, hence the discrepancy.

Jensen and Richart,<sup>(34)</sup> in their short-time tests on creep in compression, found no effect of variations in proportions of cement paste when the water-cement ratio was held constant.

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\* Specific creep is defined as creep per unit stress.

\*\* Alite, belite, celite are designations of cement clinker constituents after Tornebohm and are described by Bogue.<sup>(92)</sup> The designations are based on color, crystal shape, and birefringence.

Davis and Davis<sup>(93)</sup> are of the opinion that the water-cement ratio, considered by itself, is a lesser factor than is, for example, the grading of the aggregate. Troxell, Raphael, and Davis,<sup>(76)</sup> on the other hand, indicate that for the same aggregate-cement ratio, concrete with higher water-cement ratio exhibits greater creep (Figure 2) while, as mentioned earlier, at the same water-cement ratio, richer mixes showed more creep than leaner ones; the amount of creep being roughly proportional to the cement paste content. Obviously, if both cement content and water-cement ratio are varied, the overall effect on creep would depend on the interaction between the two trends discussed above. The higher creep in leaner mixes, as reported by Davis and Davis,<sup>(93)</sup> may be viewed in the light of the fact that leaner mixes invariably have higher water-cement ratio associated with smaller cement contents, and that the former factor predominates.

Wagner<sup>(77)</sup> plotted ratio  $c_{\infty}/c'_{\infty}$  against the water-cement ratio for a number of tests from different investigations ( $c_{\infty}$  is the limiting specific creep for a given water-cement ratio, and  $c'_{\infty}$  is specific creep for a water-cement ratio of 0.65) and obtained a good correspondence with the parabolic law suggested by Lorman.<sup>(36)</sup>

Hummel, Wesche, and Brand<sup>(94)</sup> also report an increase in creep at higher water-cement ratios.

#### b. Grading, Particle Size and Composition of Aggregate

Davis<sup>(95)</sup> reported that, other things being equal, the smaller the fineness modulus, the greater was the creep. For the same consistency of the mix, variations in grading of the aggregate had a relatively larger effect upon the magnitude of creep for leaner mixes than richer ones. More recent tests carried

out at the Waterways Experiment Station, Mississippi<sup>(96)</sup> using  $1\frac{1}{2}$  in. and 6 in. maximum size of aggregate indicated that the aggregate size, per se, might have little influence on creep but would have an indirect effect by modifying the unit paste content. For the same water-cement ratio, concrete made with 6 in. aggregate exhibited less creep than that made with  $1\frac{1}{2}$  in. aggregate. The unit paste content for the former was lower than that of the latter.

Davis and Davis,<sup>(93)</sup> on the other hand, found that an increase in the fineness modulus of the aggregate resulted in increased creep. Similar results were reported by Davis, Davis, and Hamilton,<sup>(25)</sup> who found that the creep of concrete using aggregate with a maximum size of  $\frac{3}{4}$  in. (FM 5.15) was roughly  $\frac{1}{2}$  to  $\frac{3}{4}$  of the creep of concrete using aggregate with a maximum size of  $1\frac{1}{2}$  in. (FM 5.69). They explained the difference as partly due to surface area. The smaller thickness of paste between the finer graded aggregate was considered to be partly responsible for the lower creep.

Davis and Davis<sup>(93)</sup> also found that concrete made with aggregates of different mineralogical characteristics had different creep properties. Concrete made with sandstone and basalt aggregate, for example, exhibited the greatest creep while those with limestone showed the least. Use of quartz, granite, and gravel resulted in intermediate values. The maximum and minimum one-year values differed by almost 100 per cent (Figure 4). They found that the instantaneous deformation was not a criterion for the magnitude of creep as far as difference in mineralogical characteristics of the aggregate was concerned. Similar effects of the type of aggregate on creep have been observed by Rusch, Kordina, and Hilsdorf.<sup>(97)</sup>

In general, hard dense aggregate with low

absorption and a high modulus of elasticity is considered desirable for making low creep concrete. Troxell and Davis<sup>(98)</sup> are of the opinion that for certain aggregates, creep of concrete might also be influenced by variations in crystalline slip, particle shape, surface texture, and pore structure of the aggregate. To minimize creep, they recommend aggregate well graded from fine to coarse with the largest practical maximum size, and lowest practicable percentage of voids in the mixed aggregate.

Hansen<sup>(79)</sup> is of opinion that the influence of aggregate type is transmitted to the creep behavior through the absorption characteristics, which affect the moisture diffusion in a manner similar to the loss of moisture by drying.

#### c. Entrained Air and Admixtures:

The available information on the effect of air entraining and other admixtures is scanty. Tests carried out at the Waterways Experiment Station<sup>(99)</sup> indicated an increase in creep due to entrained air. Concrete with 5.4 per cent entrained air showed 25 per cent more creep than non-air entrained concrete with 1.7 per cent air content. Jones, Hirsch, and Stephenson,<sup>(100)</sup> studied the effect of air entrainment on creep in light-weight aggregate concrete and found that the "net effect of entrained air usually is to increase the amount of creep in concrete, however, in moderate amounts, up to about five to six per cent, the effects of increasing the concrete's workability, resistance to weathering and reducing the water requirement usually outweighs the detrimental effects and this may even result in less creep."

The effect of using pozzolans to replace a part of cement is to increase creep. Washa and Fluck<sup>(54)</sup> recommend that where creep is an important factor, proprietary compounds should

not be used unless their effects on shrinkage and creep have been previously investigated, in view of their uncertain reactions when used with different cements and in different mixes.

## 2. Mixing Time and Method of Compaction

No study seems to have been made of effect of mixing time on creep of concrete. However, it would appear that the effect of an increase in mixing time would be comparable to using high early strength cement, that is, of decreasing the magnitude of creep.

Davis<sup>(101)</sup> and Thomas<sup>(24)</sup> have studied the effect of vibration on creep of concrete. Davis found creep in vibrated concrete to be less than that of unvibrated concrete while Thomas found no significant difference. Jensen and Richart<sup>(34)</sup> found that vibrated concrete showed slightly greater creep than hand-tamped concrete.

## 3. Curing History and Age at Loading

Davis, Davis, and Hamilton<sup>(25)</sup> found that creep of concrete loaded at the age of 28 days (moist cured at 70°F until the time of loading) was roughly twice that of the concrete loaded at 3 months while the creep of concrete loaded at 7 days was three times that of concrete loaded at 3 months. The effect of loading at an early age was not marked in dry storage. They also noted that the rate of creep after about one year under load was practically independent of the age at the time of loading for a given sustained stress and a given storage condition (Figure 5). This suggests that the ability of concrete to resist creep is primarily a function of the degree of hydration.

Temperature and humidity during curing affect the creep of concrete by influencing the degree of hydration. Hence, curing at higher temperatures and greater humidity would presumably be beneficial in minimizing creep.

For loading at very early ages, creep may

not be proportional to the sustained stress. Also the effect of age, at the time of loading, in reducing the rate of creep is more pronounced at higher sustained stresses than at lower ones.

Glanville<sup>(21)</sup> studied the influence of age at loading on creep. He plotted (Figure 6) creep values so that the final portions of curves for different ages were all coincident. The curves show that there is a comparatively large preliminary movement during the first month or so, followed by subsequent movements which are almost the same for all specimens of a mix and depend only on the age of concrete.

## 4. Stress: Type, Magnitude, Time-Variation and Duration

a. Type of Stress: Most of the information available in the literature on creep of concrete pertains to compressive or flexural stresses, probably because these cases are the easiest to investigate and are also the cases most frequently encountered in practice. Only limited information is available on creep under other types of stresses. These include tests under sustained tensile, torsional, biaxial, and triaxial stresses. As most of the readily available information deals with compressive stresses, creep behavior under types of stress other than compressive, only, will be discussed in this section and compared to the behavior under compressive stresses. It may be stated that, in general, the creep behavior under types of stresses other than compressive is very similar, at least qualitatively, to the behavior under compressive stresses.

1.) Creep in Tension: Creep in tension was investigated by Davis, Davis, and Brown<sup>(33)</sup> and was reported again by Troxell, Raphael, and Davis.<sup>(76)</sup> The initial creep rate was found to be larger for tensile tests, while

after a few weeks, the creep rate in compression became larger, suggesting the limiting values to be probably equal (Figure 7). In tests on unreinforced beams the tensile creep was found to be somewhat greater than the corresponding compressive creep.

Glanville<sup>(21)</sup> tested three concrete cylinders, of which one was loaded in compression, and one in tension; the third one was used as a control. He found creep in six months to be almost exactly the same under tension and compression.

Dutron<sup>(102)</sup> and L'Hermite<sup>(74)</sup> carried out experiments on creep in tension but could not obtain conclusive correlations between the magnitudes of creep in compression and tension.

2.) Bond Creep: Experiments by Shank,<sup>(103)</sup> on the study of influence of sustained loads on plain round reinforcing bars in compression splices embedded in concrete, showed movement relative to concrete without breaking the bond. He found bond creep to be independent of the distance between the bars. The observed creep was considerable in cases where high unit bond stresses were encountered; the bond creep was not found to be proportional to the compressive strength of concrete. Bond creep was found to be beneficial in eliminating stresses in the steel due to the shrinkage in the concrete. Bond creep in the case of deformed bars was not studied. No other studies appear to have been conducted on this aspect.

3.) Creep under Torsional, Biaxial and Triaxial Stresses: Anderson's<sup>(104)</sup> experiments on creep in torsion showed that creep in torsion had the same general characteristics as in compression. The results obtained were for specimens loaded at the age of 28 days, the creep measurements were taken for 105 days.

Duke and Davis,<sup>(41)</sup> conducted tests on

creep under combined sustained stresses, for various ratios of axial to lateral stress. They also conducted creep tests in torsion. They found the general creep behavior to be similar to that under uniaxial stress. They proposed a formula for correlating creep under biaxial and triaxial tests with results of test under uniaxial stress, which was reasonably close for the first month or so. They found creep under shear to be numerically equal to approximately twice the creep in compression.

Davis, Davis, and Hamilton<sup>(25)</sup> in their tests on creep under triaxial loading observed that the lateral pressures required to maintain the lateral dimension of the specimen constant, decreased rapidly in the first day or two after loading and more slowly at later ages. They also found that the axial creep occurring when the lateral dimension was maintained constant was of the same order of magnitude as the creep observed in similar specimens under no lateral restraint.

Ross<sup>(67)</sup> studied creep in concrete under biaxial stresses. He found the resultant creep in the principal directions had magnitudes generally of the same order as the corresponding creep under uniaxial stresses, and concluded that the Poisson's ratio for creep was negligible.

b. Magnitude of Stress: Creep at a given time has been found by many investigators to be proportional to the sustained stress up to a certain stress level expressed as a fraction of the ultimate strength. Although there is no general agreement on the magnitude of this limiting stress level, it is generally conceded that it is reasonably close to half of the ultimate strength of concrete at the time of loading. Thus, in the range of normal working stresses, creep may be taken to be proportional to the sustained



stress. The error involved increases rapidly as the stress level increases (Figure 8).

Creep has been observed to be present at stress levels as low as one per cent of the ultimate strength. Thus, there does not seem to be any limiting stress level or threshold below which creep is absent.

Davis, Davis, and Hamilton<sup>(25)</sup> found the stress-strain proportionality for sustained stresses to be valid for only very low stresses, the deviations being greater for wet-stored concrete and concretes loaded at the age of 7 days. They advocated a cautioned use of the proportionality relation.

Glanville<sup>(21)</sup> studied the creep which occurred during the relatively short period needed for the conventional determination of the stress-strain relation. He obtained curves for strains after periods of 5, 15, 30, and 60 seconds (Figure 9). The method of plotting was devised in order to exaggerate the form. From each value of the strain a quantity proportional to the stress was deducted. From these curves, an approximate correction to zero time was estimated, and it was found that the strain for instantaneous loading was practically in direct proportion to the stress for the various mixes studied, except in very lean mixes where the rate of creep was too rapid to be accurately recorded.

Glanville<sup>(21)</sup> also studied long-time creep of concrete having an ultimate strength of 1500 psi and subjected to a sustained stress of 600 psi; he found no appreciable departure from proportionality between stress and strains. Observed departures, if any, did not become more pronounced with time. He confirmed these findings on a richer concrete mix with an ultimate strength of 4010 psi, for stresses up to 1500 psi.

Davis and Davis<sup>(93)</sup> tentatively concluded that within the range of working stresses, the

sustained stress-creep ratio, like the instantaneous stress-deformation ratio, would tend to become constant as the age is increased.

Tests by L'Hermite<sup>(74)</sup> showed a proportionality up to a stress of 30 per cent of ultimate strength, there being a marked deviation at higher levels. Freudenthal and Roll<sup>(105)</sup> report proportionality between creep and stress up to a stress level of about 23 per cent.

Ross<sup>(89)</sup> observed creep strains proportional to the stress in portland blast-furnace slag cement concrete up to stresses around 30 per cent of the ultimate strength.

The problem of creep behavior under high loads was studied by Shank<sup>(106)</sup> who found the proportionality limit to extend up to a stress level of about 40 per cent.

Jones and Richart,<sup>(107)</sup> in their very short-duration creep tests, found that the secant modulus at 50 per cent load level was not much different from the initial tangent modulus. An increase in the rate of loading tended to cause the secant modulus at 50 per cent level to increase, while the secant modulus at 90 per cent level showed a consistent increase.

Jensen and Richart,<sup>(34)</sup> in their short-time creep tests, found the limit of proportionality at a stress level of about 50 per cent. Beyond this they found creep to increase at a rapidly increasing rate.

It may thus be observed that there is little agreement on the above point. The observed limit for sustained stress up to which creep strains are in linear proportion has been reported by different investigators, all the way from 0.25 to 0.60 of the ultimate strength at the time of loading. However, creep strains are commonly expressed in terms of strains per unit stress. Most mathematical

relations for expressing the creep strain as a function of time assume proportionality and do not contain any term for stress, Straub's expression<sup>(22)</sup> being an exception. In view of insufficient information available about the stress-strain relationship in creep, no formula expressing the deviation from proportionality seems to have been proposed. For most purposes it is considered reasonable to assume that the proportionality law applies within the usual limits of the working stresses.

c. Duration of Stress: It may be stated in general that the greater the duration of the stress, the greater the resultant creep. Also that the rate of creep decreases progressively with time. The last statement is obviously not applicable to stress levels high enough to produce a creep failure. The actual functional relationship, as represented by the creep-time curve under sustained stress, is still a matter of controversy, and is discussed in more detail elsewhere in the report. Opinion is divided on the question of the existence or non-existence of a limiting magnitude of creep. Results of tests reported by Troxell, Raphael, and Davis<sup>(76)</sup> indicate that measurable creep could be observed in concrete specimens under sustained compression, even after 20 years, although the rate was very small. Figure 10 depicts the trends observed. Similar observations on creep in arches and cooling towers over periods of 10 to 15 years have been reported by Valette and Kahn.<sup>(108)</sup> Flugge,<sup>(55)</sup> in his analysis of test data from the studies by Glanville and Davis, concludes that after about 400 days the creep rate assumes an almost constant value, which might be construed as implying the absence of any defined limit for creep.

In practice, however, it is usual to

assume that most of the creep deformation is completed within a period of 4 to 5 years. This may be a reasonable assumption for normal structural members, but may have to be considered in the case of mass concrete structures. Little information is available in the literature on this aspect.

#### d. Time-Variation of Stress:

Relatively little information is available on creep behavior of concrete under time variation of stress. The superposition principle, as outlined by McHenry<sup>(39)</sup> tacitly implies the complete recovery of both the instantaneous and time-dependent strains. The effect of time variation of stress can then be accounted for by representing the stress history as a system of individual stresses of various magnitudes and signs applied at suitable instants and sustained thereafter. The actual recovery is, however, only partial -- which introduces limitations on the superposition principle. Another interesting phenomenon has been reported by Probst,<sup>(109)</sup> wherein the creep strains, under a rapidly fluctuating stress applied for a given duration, decreased as the frequency of the fluctuation was increased. The elastic deformations were not seen to be frequently dependent within the range employed. Also uniform fluctuations were stated to cause less creep than irregular patterns of fluctuations of comparable range. L'Hermite,<sup>(74)</sup> too, supports the view that under rapidly fluctuating stresses, inelastic deformations are considerably reduced and the material exhibits a predominantly elastic behavior.

#### 5. Strength of Concrete.

Strength of concrete is the result of the interaction of a large number of factors, which also influence the creep behavior of concrete, although not necessarily in the same manner or to the same extent. The influence on creep of

water-cement ratio, cement content, degree of hydration, and characteristics of the ingredients have already been discussed above under the relevant topics. To recapitulate, it may be stated that for given characteristics of the ingredients and under identical conditions of environment and stress, creep increases with increase in the water-cement ratio and cement-aggregate ratio, and decreases with increase in the degree of hydration or age. In varying the strength of concrete mixes, it is usual to adjust the water-cement ratio and cement-aggregate ratio in opposite directions. The net effect on creep would thus be determined in any particular case by the resultant interaction of two opposite trends. The above effect may be analyzed either on the basis of a given absolute stress or a given stress-strength ratio, i.e., stress level.

Considering a given magnitude of stress, the influence of variation of the water-cement ratio predominates, masking the effect of cement content, at least within the usual range of mixes. This is indicated by the larger creep strains in leaner mixes and vice versa, when both water-cement ratio and cement-aggregate ratio are varied while maintaining a comparable workability, as observed by Davis and Davis<sup>(93)</sup> and others.

If, however, stress-level or the ratio of applied stress to the concrete strength is considered as the relevant parameter, then the effect of water-cement ratio is more or less eliminated. According to Lyse,<sup>(110)</sup> creep in concrete at a given stress level is in approximate proportion to the percentage of cement paste in the mix, irrespective of its water-cement ratio or strength. Similar observations have been made by Neville,<sup>(88)</sup> Jensen and Richart,<sup>(34)</sup> on the other hand, report almost no effect of cement paste content for the same water-cement ratio.

According to the above, for a given stress level leaner mixes would be expected to exhibit less creep than richer ones, due to the smaller percentage of cement paste in the former. This has been corroborated by many investigators. For very rich mixes, however, Freudenthal and Roll<sup>(105)</sup> report a tendency towards decrease in creep.

#### 6. Moisture Condition of the Specimen, Member or Structure

The effect of moisture content of the concrete on its creep is, in general, difficult to isolate from associated influences of moisture movement, nonuniform shrinkage, and continuing hydration during the period required to attain moisture equilibrium with the environment. The time required for such equilibrium is, in most cases, quite considerable.<sup>(79)</sup>

For fully mature concrete in moisture equilibrium with the surroundings it may, however, be stated that a higher moisture content results in higher creep. Davis, Davis, and Hamilton<sup>(25)</sup> compared the creep of dry lacquered and saturated concrete. The latter showed creep strains almost twenty times more than the former under load sustained for a year. Troxell, Raphael, and Davis,<sup>(76)</sup> on the other hand, observed no correlation between creep and total water content of the concrete. The very narrow range of water content in the mixes studied may have been responsible for the observation. Glucklich and Ishai,<sup>(111)</sup> using well cured mortar specimens under torsional loads, demonstrated the reduction of creep with decrease in the evaporable moisture content. Fully dry specimens exhibited almost no creep.

#### 7. Hygrostatics or Moisture Movement

The process of drying out has been found to result in increased creep.<sup>(79)</sup> The effect of absorption of moisture on creep, however,



does not seem to have been studied in any significant detail. It is therefore difficult to state whether the observed influence is a result of movement of the moisture alone, or also includes the effect of nonuniform shrinkage during drying out.

#### 8. Storage Humidity and Temperature

For originally saturated specimens exposed to atmospheres at constant temperature but different humidities, creep has been found to increase with a decrease in the humidity of the environment. Troxell, Raphael, and Davis (76) report creep strains in an atmosphere at 70 per cent relative humidity as roughly twice, and those at 50 per cent to be  $2\frac{1}{2}$  to 3 times, the strains at 100 per cent relative humidity (Figure 11). The temperature was maintained at 70°F. The actual ratios depended on the age at loading. For specimens brought to moisture equilibrium with the environment and then sealed, the behavior was contrary to the effect described above for exposed specimens, i.e., creep increased with increasing humidity. It would thus appear that the humidity of the environment as such has little significance. It is the difference between the hygrometric condition in the interior of the specimen and the external atmosphere which is important. The resultant increase in the rate of drying of moist specimens exposed to atmospheres of low humidity, together with the consequent hygroscopic imbalance in the specimen, may thus be considered as the major factors responsible for increased creep.

Temperature of storage influences creep directly, resulting in higher creep strains at higher temperatures. (32, 28, 112) In case of partially cured concrete, the temperature of the environment can also affect the rate of hydration and thus exercise an indirect influence on creep.

#### 9. Coexistence of Shrinkage

The phenomena of shrinkage and creep are influenced by many common factors in a surprisingly similar manner and can seldom be completely isolated. In addition, there appears to be significant interaction between the associated deformations which makes it all the more difficult to isolate the two effects. The coexistence of shrinkage almost always causes an increase in the resultant creep strains. Pickett (38) explains the interaction in terms of nonuniform shrinkage stresses and increasing nonlinearity of creep strains with increased stress levels. Maney (37) maintains that creep, as observed, is entirely due to a gradual release of restrained shrinkage at the surface under applied load and that without shrinkage no creep strains can be exhibited. The argument has, however, been questioned. (113) The effect of shrinkage on creep is well recognized but quantitatively it still remains indeterminate.

#### 10. Size and Shape of the Specimen

Many investigators have reported a decrease in creep with increase in size of specimen. Troxell, Raphael, and Davis (76) studied cylindrical specimens, cured and stored at 100 per cent relative humidity and 70°F temperature. Ten-in. diameter specimens showed 30 per cent less creep strain than 6-in. diameter specimens, after  $9\frac{1}{2}$  years under load. Most of the difference, however, was developed very early during the load history (Figure 12). Delarue (108) studied prismatic specimens with a square cross section. Specimens with a 17-cm side exhibited creep strains 27 and 17 per cent less than those with a 12-cm side, after 28 days and 400 days, respectively. The major divergence was observed to occur at an early stage and only the absolute difference appeared to be effective later.

The effect is usually explained in terms of the increased seepage path to the free surface in the case of larger specimens.<sup>(25)</sup> On the contrary, there is evidence to indicate that the loss of moisture to the environment associated with creep is insignificant in comparison to the gross change in volume.<sup>(74, 108)</sup> Increased creep under conditions of drying is also well known. These contradicting observations point towards a complex interaction of several factors which manifest themselves as a resultant size effect. Differences in shrinkage and hydration may also play a significant role. Neville<sup>(69)</sup> suggests that creep might be partly viscous and partly due to seepage and that the latter alone is influenced by humidity and temperature conditions while the former is not so influenced. The size effect may thus be considered as associated with the interrelated phenomena of seepage and shrinkage which influence creep only to a partial extent. A systematic investigation of the effect of specimen size and shape on creep is underway at the Portland Cement Association Research and Development Laboratories.<sup>(114)</sup>

#### 11. Presence of Reinforcement

The presence of reinforcement in concrete

tends to counteract the time-dependent deformations. An increasing share of the external load is transferred to the reinforcement and the concrete is progressively relieved of its stress as creep progresses. The decreasing concrete stress in turn rapidly reduces the creep rate. The behavior may be roughly visualized by imagining the concrete to possess a time-decreasing elastic modulus. This approach is in fact most commonly used to estimate the effect of creep on the behavior of reinforced concrete members by the "sustained" or "effective" modulus method.<sup>(13)</sup>

More sophisticated analyses which attempt to estimate creep strains under variable stress have been developed.<sup>(115)</sup> A linear rheological model, analogous to a reinforced concrete member has been proposed by Ross<sup>(40)</sup> for analyzing the effect of creep and shrinkage on axially loaded columns. Freudenthal,<sup>(73)</sup> Torroja, and Paez<sup>(66)</sup> have presented nonlinear models. Their numerical analyses is applied to prestressed and reinforced concrete members. England and Ross<sup>(116)</sup> have used a simple step-by-step procedure for analyzing the time-dependent stress distribution in reinforced concrete sections subjected to thermal gradients.

### III. CREEP AND STRUCTURAL BEHAVIOR

#### A. GENERAL CONSIDERATIONS

It is the effect of creep on the strength and behavior of concrete structures that is of primary concern to the structural engineer, and it is this aspect of creep that makes its study so important from a practical point of view. In most cases, a general qualitative analysis is fairly easy to make, but rigorous quantitative analyses appear to defy all attempts. The association of creep with elastic deformation, its dependence on the size of specimen, lack of information on creep magnitudes under variable loadings, and nonexistence of any general method of quantitative estimation of creep under variable stresses from constant stress data constitute some of the factors which make it extremely difficult to estimate quantitatively the effects of creep in a given structure. A qualitative idea is, of course, of considerable help to the engineer in assessing the importance of creep in a given structure and, if necessary, in taking precautions to reduce the amount of creep by careful quality control, increasing the age at loading, improving curing conditions, controlling drying out, or other such possible methods.

Creep may be desirable or undesirable depending on where it occurs. In statically indeterminate structures, creep relieves stresses from the highly stressed parts and hence is desirable. On the other hand, in prestressed concrete members creep causes a

loss of prestress. Creep results in excessive deflections in reinforced concrete beams and slabs, which makes it undesirable.

As Freudenthal<sup>(73)</sup> put it, creep sensitivity is dependent on two factors, namely the ratio of the sustained stress to the total stress and the ratio of the total stress to the ultimate strength of the concrete. The first he calls a measure of the structural component of creep sensitivity depending on the type of load, span, etc. The second he refers to as the material component depending on the degree of utilization of the material. The larger these are, the more important the effect of creep becomes.

Sustained stresses may be induced either by sustained forces (load-stresses) or from enforced or restrained deformation (deformation stresses). Creep affects only the distribution of load stresses, but determines the magnitude of deformation stresses. In statically indeterminate structures, both occur, hence they are more creep-sensitive. Simple prestressed structures are internally indeterminate. Strictly speaking, all reinforced structures are indeterminate.

#### 1. Reinforced Concrete Beams and Slabs

In a statically determinate reinforced concrete structure subjected to a sustained load, deflections increase at a decreasing rate with time; the deflection-time curve shows a great similarity to the creep-time curve for concrete in compression. The compressive

strains in the concrete increase rapidly with time but the deflections increase at a smaller rate. For example, in a series of experiments by Washa,<sup>(47)</sup> reinforced concrete slabs, 3- by 12-in. in cross section, were subjected to sustained loads for 5 years. A 7.42-in. deflection was observed at the end of five years in a 12½-ft. simple span, the total deflection being 34 per cent due to initial loading, 20 per cent due to warping caused by shrinkage, and 46 per cent due to creep, while the corresponding compressive strain was about six times the strains at seven days. The reason why the compressive strains increase faster than the deflection is that as the compressive strains increase, the neutral axis shifts down due to the tensile stresses not increasing in proportion; as a result of which, the corresponding increase in rotation per unit length or in the deflection, is relatively smaller.

The lowering of the neutral axis causes the center of compression to come closer to the center of tension and the somewhat decreased lever arm results in a slight increase in the steel stress. The increase, however, is relatively small.

In a beam or slab with compression reinforcement, creep deflections and compression strains are much less, as the compression reinforcement relieves the concrete stresses and thus tends to counteract the creep strains. Investigations, conducted by Washa and Fluck,<sup>(117)</sup> showed a considerable effect of using compressive reinforcement in arbitrary amounts. They observed that the creep deflections were reduced to one-half by inclusion of compressive reinforcement equal in amount to the tensile reinforcement. Likewise, they also observed the effect of using a small span length-to-depth ratio in reducing the creep deflections; for reduction of the ratio by a factor

of 3.5, the observed creep deformations were reduced by a factor of four to six.

The ultimate strength of a beam or a slab is not significantly affected by creep. The creep strains occurring near the ultimate loads are very rapid, which cause a redistribution of stresses, thus eliminating the effects of stresses induced by creep at lower loads.

## 2. Reinforced Concrete Columns

In reinforced concrete columns under sustained loads, the gradual creep of concrete causes the reinforcing steel to bear an increasingly greater share of load, until strains in concrete and steel become equal to the limiting creep of the concrete under the load then shared by it. The more the concrete creeps, the more the load is transferred to steel, unless the steel yields. The creep strains, in a reinforced concrete column, are therefore much smaller than those in a corresponding plain concrete column of the same cross sectional area loaded with the same initial load, the difference depending on the percentage and type of steel used as reinforcement. The reinforcing steel may be subjected to a stress increase of up to three to four times the initial stress, as a result of creep in the surrounding concrete.

Richart and Staehle<sup>(118)</sup> found that a considerable period of sustained loading at moderate stress levels did not affect the ultimate strength of reinforced concrete columns to any significant degree. Creep under working stresses or shrinkage thus appears to have little effect on the ultimate strength of a reinforced concrete column, as determined by a conventional short time test. The much higher strains associated with near ultimate loads seem to mask, almost completely, any effects of creep under relatively low sustained loads, or of shrinkage.

### 3. Statically Indeterminate Structures

As mentioned earlier, statically indeterminate structures are subject to both load stresses and deformation stresses, a circumstance which makes them more sensitive to creep. In statically determinate structures, the principal effect is the increased deflection, accompanied by a slight redistribution of stresses across the section, while in indeterminate structures, the ultimate strength may also be significantly affected due to redistribution of stresses along a member or between individual members.

In two-span continuous beams tested by Washa and Fluck,<sup>(119)</sup> they found that at the sections of the maximum positive moment, the deflection and compressive and tensile creeps were similar to those in a simple beam. However, the one-year values of deflection and compressive creep in the continuous beams were 30 to 50 per cent and 60 to 80 per cent, respectively, of the corresponding values in simply supported beams. At the center support the tensile and compressive creep increased at a decreasing rate. The tensile creep was found to be less for beams with compression steel in the sections of positive moment. The center reaction also increased by 1 to 5 per cent of the initial. In addition, the support moment increased and the numerically smaller midspan moment decreased, which was in apparent contradiction with the expected behavior if moment redistribution were to occur. This could be explained, however, by the fact that the negative moment region had a relatively heavier reinforcement than the positive moment regions, resulting in more creep at the latter regions.

Usually, however, creep does not cause significant redistribution of moments in continuous beams unless the supports yield. Shrinkage, however, tends to cause an increase

in the support moments in continuous beams. In cases where a support does yield, moments proportional to the product of the modulus of elasticity and the moment of inertia are generated at the supports. These moments tend to be redistributed due to creep; in a two span continuous beam, for example, this would result in a gradual increase in the center support reaction.

In reinforced concrete arches, stresses induced by yielding of supports are gradually relieved due to the effect of creep. The more severe the overstress at a point, the more rapidly will it be relieved.

In arches, the mid-span deflections tend to increase with time as a result of creep. Deflection measurements on a full-sized reinforced concrete arch were made by Glanville who found a progressive increase in the deflections as a result of shrinkage and creep over and above the daily and seasonal fluctuations. Shrinkage tends to decrease both the span and the rise while creep increases the span and reduces the rise.

Similar observations are reported by Vallette.<sup>(108)</sup>

The effect of creep on plain and reinforced concrete arches was studied by Whitney.<sup>(120)</sup> He found in his investigation that for live loads, the action of arches conformed closely to expected elastic behavior. Some of his further findings were as follows: Initial stresses produced by flexure at an early age are largely eliminated by creep. The earlier the decentering, the more the stress carried by steel and the less the effect of abutment yield. Stresses due to high temperatures during setting have no appreciable creep effect on a reinforced concrete arch. Due to large deformations produced by creep, shrinkage, and temperature variations, high stresses evidently exist in the steel in addition to live

load stresses.

#### 4. Creep in Prestressed Concrete

In prestressed concrete construction, the presence of creep is particularly conspicuous and undesirable. Creep and shrinkage, in this type of construction, are responsible for much of the loss of prestress, especially in pre-tensioned members. As a matter of fact, it is mainly this loss of prestress that makes it imperative that high strength steel be used as reinforcement in prestressed construction so that even after a considerable loss in stress, a large enough stress is retained. All the loss, however, is not due to creep and shrinkage in concrete, but significant losses are due to elastic shortening and relaxation in steel.

It is, therefore, desirable that for prestressed concrete construction special precautions be taken to avoid excessive creep and shrinkage in concrete, which can be achieved by adequate quality control, use of high early strength concrete, steam curing, increasing the age at transfer of stress, etc. The loss due to creep and shrinkage of concrete, for an initial steel stress of about 125,000 psi, could be about 30,000 psi. This is a large percentage of the initial stress; the significance of creep and shrinkage in prestressed concrete construction is obvious.

#### 5. Creep in Mass Concrete

In structures such as concrete dams, thick retaining walls, bridge piers, etc., where the bulk of concrete is so thick all around that the heat generated by the exothermic reaction of the hydration of cement is not quickly dissipated, cracks are observed in many cases. There is a thick core which is at a relatively higher temperature than the rest of the concrete, and hence expands, producing tension on the outside and compression in the core. Since the concrete is young at this

stage, the stress is very quickly relieved due to creep. As the interior gets cooler it produces compression on the outside and tension inside, both of which again tend to be relieved, but not as fast as before due to the increased age of concrete. This is probably responsible for observed distress. The severe temperature differences that are produced in massive structures can be avoided by the use of low heat cement, low water-cement ratio and lower cement contents, low placing temperatures, controlled rate of concrete placement, or by an internal cooling system in extreme cases.

In general, creep of mass concrete helps in reducing high stress concentrations generated by temperature variation. Slow hardening cements are advantageous in mass concrete. Low-heat and pozzolanic cements creep relatively more and cause greater stress relief and consequently reduce distress caused by temperature variation. Shrinkage cracks tend to appear in a mass concrete structure on the outside. Due to low permeability, the interior remains moist while the outer layers tend to shrink, and are consequently under tension. Creep in the concrete, which is rapid at an early age, tends to relieve these shrinkage stresses and reduces the extent of shrinkage cracking.

It has been observed that other things being equal, creep in larger specimens is less than creep in smaller ones. The observed creep in mass concrete, although smaller than that in laboratory specimens, is not as low as might be expected on the basis of extrapolation from laboratory tests on specimens of various sizes.<sup>(96)</sup>

#### B. CREEP AND STRUCTURAL ANALYSIS

As has been pointed out earlier, a quantitative analysis of the effect of creep on the behavior of concrete structure poses a



rather formidable problem. The difficulties encountered may be classified into two major groups. The first comprises the relative inadequacy of the present extent of our knowledge about the quantitative relationships between the creep of concrete and the various factors which influence it. The second consists of the difficulties involved in the analytical procedures themselves, which could become prohibitively complicated for all but the most simplified approaches. Special difficulties are encountered in case of structures where the stresses are strain dependent, in the sense that stress under a given load cannot be computed without a knowledge of the absolute deformations of the structure or structural member. Typical examples of such cases are reinforced and prestressed concrete members, thin arches and shells, and members supported on elastic foundations. Serious difficulties are met when creep strains vary nonlinearly with applied stresses, as in the analysis of structural behavior near ultimate loads.

Several empirical and semi-analytical procedures are available which can, within their inherent limitations, provide fairly satisfactory estimates of the effect of creep for use in practical structural design. Most of these procedures are limited to cases where the stresses are low enough to warrant the assumption of a linear relationship between the applied stress and the resultant creep strain at a given time, thus permitting the use of the principle of superposition. No satisfactory generalized approach is available for analyses involving nonlinear creep behavior, although certain numerical and approximate techniques have been proposed for the solution of certain specific problems.

Some of the principal procedures, their applicability and limitations are briefly dis-

cussed.

#### 1. Effective Modulus Method

The almost linear relationship between the applied stress and the consequent creep strain at a given time, for relatively low stresses, forms the basis for this approach. The procedure consists of using a reduced "effective" value of the elastic modulus for concrete in the conventional formulas employed for elastic analysis. Although strictly applicable only to cases where the stress distribution is not time-sensitive, the method is nevertheless very widely utilized for the analysis of reinforced concrete structures, and is perhaps the simplest technique available for routine work.

The effective modulus is introduced by increasing the usual modular ratio  $n$  by a factor ranging from 2 to 4 and by using this modified value for the computation of concrete and steel stresses and the corresponding transformed areas and moments of inertia employed for estimating deformations, under sustained loads.

Faber,<sup>(13)</sup> in 1927, first suggested the use of a reduced elastic modulus for concrete in order to allow for the effect of time-dependent strains on the behavior of reinforced concrete members under sustained load. Further refinements were introduced by Shank,<sup>(29)</sup> Glanville,<sup>(35)</sup> and others, who proposed simplified formulas based on empirical unit creep-time curves, for the analysis of structural concrete under various modes of loading.<sup>(115)</sup>

As mentioned above, the use of an "effective" or "sustained" modulus implicitly disregards the influence of the previous stress history on the creep strain at a given time. While this assumption appears to result in reasonably accurate analysis in the case of constant loads, its applicability to cases

involving variation of applied load or deformation is questionable.

## 2. The Superposition Approach

A very powerful concept has been outlined by McHenry,<sup>(39)</sup> which forms the direct or indirect basis for most procedures developed for the analysis of creep of concrete. The principle of superposition states that the strains produced in concrete at any time  $t$  by a stress increment applied at time  $t_0$  and sustained thereafter is independent of the effect of any stresses applied earlier or later than  $t_0$ . The stress increments may be positive or negative. Removal of a given load may thus be considered as equivalent to the application of an equal but opposite stress (Figure 13). A complete recovery on removal of load is implied, which may perhaps be questioned. Within this limitation however, the validity of the superposition principle is well supported by experimental evidence, up to stress levels corresponding to those permitted under working loads in conventional design practice. The validity of the principle is not influenced by variations in the properties of concrete with time, as long as variations affect the entire material similarly.

In principle therefore, if creep-time relationships for unit stress applied to a given concrete at various ages are available, the strain at any given time due to a given stress history can be derived by summing up the effects of individual positive or negative increments. The basic creep data may be expressed as a family of specific creep-time curves (Figure 14), or in a three-dimensional representation as a specific creep surface (Figure 15). The converse problem of deducing the stresses at any given time due to a given strain history can also be solved using the same basic creep data. It may be observed that for cases involving constant stresses,

the principle of superposition reduces to the "sustained" or "effective" modulus approach.

McHenry proposed the following empirical unit creep-time relationship, based on the assumption of a finite potential creep, proportional to the applied stress and a rate of creep proportional to the remaining potential creep at any instant.

$$c = A(1 - e^{-rt}) + B e^{-pk}(1 - e^{-qt})$$

where

$c$  = creep due to unit stress or specific creep

$k$  = age at time of loading

$t$  = time after application of load

$A$ ,  $B$ ,  $p$ ,  $q$ , and  $r$  are constants to be determined experimentally

The second term in the above expression represents the effect on creep of the variation in the properties of concrete with time, before full maturity is attained.

The expression may be simplified by assuming equal values for the constants  $r$  and  $q$ , as under:

$$c = (A + B e^{-pk})(1 - e^{-rt}).$$

The above expression is claimed to represent fairly well the creep-time relationship for concrete from an age of five days to one year. For a given stress history with the stress  $\sigma = 0$  at  $k = T_0$  and varying continuously between  $k = T_0$  and  $k = T$ , the creep strain  $\epsilon_c$  at time  $T$  is thus given by:

$$\epsilon_c = \int_{T_0}^T [1 - e^{-r(T-k)}] [A + B e^{-pk}] \frac{d\sigma}{dk} dk.$$

The above integral may be evaluated by rigorous or numerical procedures, and is applicable, within the limits of the assumptions made, to stresses resulting from any cause. Deformations due to unrestrained response to temperature, shrinkage or other volume change effects are, however, excluded and must be added to the



computed creep strains to obtain the total deformation. Conversely, unrestrained deformation must be deducted from the total deformation before computation of resultant stress.

### 3. The Elastic-Viscoelastic Analogy

A very useful development of the superposition principle is the so-called "elastic-viscoelastic" analogy, for the analysis of cases involving time variation of applied stresses or strains. The application is, however, limited to cases where the stresses and strains are not interdependent in the sense that the elastic stresses under a given applied load or the strains under a given applied deformation are independent of the absolute value of the elastic constants of the material. The analogy makes use of fictitious magnitudes of applied load or deformation, which when used with a conventional elastic analysis yield the required solution for resultant strains or stresses corresponding to the original time variant pattern of applied load or deformation.

The development of the analogy after McHenry<sup>(39)</sup> based on generalized creep and relaxation functions is briefly outlined.

The specific creep  $c$  may be expressed as a general function of the time  $k$  at which the unit stress is imposed and  $(T-k)$ , the elapsed time.

$$c = \psi(k, \overline{T-k}).$$

Then the total strain  $\epsilon_T$  at time  $T$ , under a time-variable stress  $\sigma(k) = 0$  at  $k = T_0$  and varying continuously to  $k = T$ , is given by:

$$\epsilon_T = \frac{\sigma_T}{E} + \int_{T_0}^T \psi(k, \overline{T-k}) \frac{d\sigma}{dk} dk \text{ or}$$

$$\epsilon_T = \frac{1}{E} \left[ \sigma_T + E \int_{T_0}^T \psi(k, \overline{T-k}) \frac{d\sigma}{dk} dk \right]$$

where  $E$  is the usual elastic modulus.

$$\left[ \sigma_T + E \int_{T_0}^T \psi(k, \overline{T-k}) \frac{d\sigma}{dk} dk \right]$$

may now be considered as a fictitious stress  $\sigma'_T$ , equivalent to the particular time variant stress pattern  $\sigma(k)$  under consideration. Thus we have

$$\epsilon_T = \frac{\sigma'_T}{E}$$

which is analogous to Hooke's law in elastic analysis.

In a similar manner, if  $\phi(k, \overline{T-k})$  is a general relaxation function corresponding to a unit applied strain, a fictitious strain  $\epsilon'_T$  equivalent to an applied time variant applied strain  $\epsilon(k)$ , may be derived as follows:

$$\epsilon'_T = \epsilon_T + \frac{1}{E} \int_{T_0}^T \phi(k, \overline{T-k}) \frac{d\epsilon}{dk} dk$$

from which  $\sigma_T = E \epsilon'_T$ .

The analogy may be expressed in an alternative form<sup>(121, 122)</sup> using operator notation as follows.

$$\text{Let } C(\sigma_T) = \frac{\sigma_T}{E} + \int_{T_0}^T \psi(k, \overline{T-k}) \frac{d\sigma}{dk} dk$$

where  $C$  may be called the creep or compliance operator whence  $\epsilon_T = C(\sigma_T)$ . The method may be extended to the analysis of creep in three dimensions by introducing an additional operator  $N$  such that at any time  $k$ ,

$$\epsilon_x = c(\sigma_x) - N(\sigma_y + \sigma_z)$$

with similar expressions for  $\epsilon_y$  and  $\epsilon_z$ .

Also

$$\tau_{xy} = 2(C+N) \tau_{xy}$$

with similar expressions for  $\tau_{xz}$  and  $\tau_{yz}$ , where  $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz})$  and  $(\epsilon_x, \epsilon_y, \epsilon_z, \tau_{xy}, \tau_{xz}, \tau_{yz})$  are the normal and shearing stresses and strains, corresponding to the coordinate directions  $x, y$ , and  $z$ . The

operators C and N are analogous to the elastic constants, as follows:

C corresponds to  $\frac{1}{E}$

N corresponds to  $\frac{\mu}{E}$

and  $2(C+N)$  corresponds to  $\frac{2(1+\mu)}{E}$  or  $\frac{1}{G}$ .

A relaxation operator R may be introduced in a similar manner such that

$$R(\epsilon_T) = E\epsilon_T \int_{T_0}^T \Phi(k, T-k) \frac{d\epsilon}{dk} dk$$

where

$$\sigma_T = R(\epsilon_T).$$

The analogy can be used conveniently for analyzing the time-dependent effects of external loads, support displacements, temperature changes, or shrinkage, which vary with time in a prescribed manner. Application of the relevant creep or relaxation operator furnishes the equivalent load, displacement, temperature, or volume change, which is then used to obtain the required solution through conventional methods of elastic analysis.

#### 4. The Rheological Model Approach

Instead of basing the analysis on a creep or relaxation function, the behavior of a suitable rheological model could be utilized instead. The idealized model elements not only facilitate the mathematical manipulation, but allow one to visualize the mechanism of the behavior to a certain degree; this advantage is not afforded by an abstract mathematical function.

Some typical examples of this approach are briefly reviewed:

a. Ross<sup>(40)</sup> used a simplified three-element model (Figure 16) with linear elements. Irrecoverable strain is simulated by a one way frictional restraint acting on the Kelvin spring during recovery only. Figures 17 and 18 show the response of the model under one basic "load-hold-unload" cycle and its repeated

application. The behavior is a fairly good representation of the observed response of concrete. Effect of shrinkage may be represented by an equivalent compressive stress applied across the Kelvin body.

The behavior of the model under the basic load cycle may be expressed by the following relations:

$$\epsilon_e = \sigma \alpha_e$$

$$\epsilon_c = \sigma \alpha_c (1 - e^{-\frac{\Phi t}{\alpha_c}})$$

and

$$\epsilon_r = \alpha_c (\sigma - q) e^{-\frac{\Phi(t-t_1)}{\alpha_c}} - \sigma e^{-\frac{\Phi t}{\alpha_c}} + q$$

where:

$\epsilon_e$  = instantaneous or elastic strain, on application of stress " $\sigma$ "

$\epsilon_c$  = creep strain at time t after application of stress " $\sigma$ "

$\epsilon_r$  = recovery strain at time t, after application of stress " $\sigma$ " and its subsequent removal at time  $t_1$

$\alpha_e$  and  $\alpha_c$  are the compliances of the free spring and Kelvin spring respectively

$\Phi$  = fluidity of the Kelvin dash pot

q = frictional restraint on Kelvin spring, during recovery

s = equivalent shrinkage stress.

On the basis of the above model, solutions are developed for the time dependent stresses in axially loaded structural members of plain, reinforced, and prestressed concrete, including the effects of creep and shrinkage. The solutions may be expressed either in terms of the model parameters or the phenomenological characteristics of the concrete.

a. Plain Concrete - stress relaxation under constant strain

$$\sigma_t = \frac{1}{\alpha_e + \alpha_c} \left[ \sigma_0 \alpha_e - s \alpha_c - \left[ (\alpha_e + \alpha_c) / \alpha_e \alpha_c \right] \Phi t \right] + (\sigma_0 + s) \alpha_c \cdot e^{-\frac{\Phi t}{\alpha_c}}$$

where:

$\sigma_0$  = the instantaneous stress ( $t = 0$ )  
corresponding to the sustained  
strain

$\sigma_t$  = the residual stress after time  $t$ .

Solutions for no shrinkage or only shrinkage correspond to  $s = 0$  or  $\sigma_0 = 0$  respectively.

b. Reinforced Concrete - change in steel stress under sustained load

The reinforcement may be represented by additional springs of equivalent compliance  $\alpha'_s$  in parallel with the concrete model. For convenience, compliances corresponding to unit load instead of unit stress are used. The following expression is obtained:

$$\Delta F = \frac{\alpha'_s \alpha'_c \left[ P + S \left( 1 + \frac{\alpha'_e}{\alpha'_s} \right) \right]}{(\alpha'_s + \alpha'_e + \alpha'_c) (\alpha_e + \alpha_s)} (1 - e^{-\beta \phi t})$$

where

$$\alpha'_e = \frac{\alpha_e}{A_c}, \alpha'_c = \frac{\alpha_c}{A_s}, \alpha'_s = \frac{\alpha_s}{A_s}, A_c \text{ and } A_s$$

being the concrete and steel areas respectively,  
and

$$\beta = \frac{\alpha'_s + \alpha'_e + \alpha'_c}{\alpha'_e \alpha'_c + \alpha'_s \alpha'_c}$$

$P$  = total load on the member

$\Delta F$  = change in the total tensile force in the reinforcement at time  $t$ .

c. Prestressed Concrete - residual prestress after a given time

Using as a basis the same model as in b above, but with the reinforcement under initial tension, a similar analysis yields:

$$F_t = \frac{F_0 (\alpha'_s + \alpha'_e + \alpha'_c e^{-\beta \phi t}) - S (\alpha'_c - \alpha'_c e^{-\beta \phi t})}{(\alpha'_s + \alpha'_c + \alpha'_e)}$$

where

$F_0$  = total tensile force in steel at transfer

$F_t$  = total tensile force in steel after time  $t$ .

The expressions given in a through c above may also be easily expressed in terms of the following more familiar parameters:

$\frac{1}{\alpha_e} = E_c$ , the elastic modulus of concrete

$\frac{1}{\alpha_c} = c_\infty$ , the ultimate creep strain under unit stress

$\frac{1}{\alpha_s} = E_s$ , the elastic modulus of steel.

The parameter  $\phi$  has no simple counterpart.

The following approximate values of the model parameters are reported for a typical 1:2:4 concrete, loaded at 25 days and stored at a relative humidity of 65 per cent.

$\alpha_e = 0.25 \times 10^{-6}$ ,  $\alpha_c = 0.5 \times 10^{-6}$ ,  
 $\phi = 1.0 \times 10^{-6}$ , and  $s = 400$ , all in pound-inch-year units.

The reference includes illustrative examples of analysis for the cases discussed above. In principle, the analysis could be extended to reinforced and prestressed concrete members under flexure and combined flexural and axial loads. The simultaneous satisfaction of the equilibrium and compatibility requirements would, of course, be more involved than that for axial loading.

b. A more detailed model (Figure 19) has been proposed by Freudenthal, (73) which attempts to represent most of the behavioral aspects of concrete, both at low stress levels where the response is essentially linear with stress and at high levels where nonlinearity sets in. A rigorous analysis of the generalized model is prohibitively involved. Under certain conditions, however, the model may be simplified for practical purposes to a linear 4-element, 3-element, or even 2-element model, depending on the stress levels and time scales involved, in relation to the characteristics of the model parameters.

Possible simplifications may thus be made if the duration for which the applied load or deformation is sustained is either much longer or much shorter than the retardation time of any part of the model. Then the time variation of its response may be disregarded in the first case and the time variation of its properties disregarded in the second. For example, as  $\tau_K$ ,  $\tau_1$ , and  $\tau_2$  are short in relation to  $\tau_M$ , the time variation of the deformation of the former need not be considered when analyzing a load sustained for a long time. The response of only the Maxwell element need be analyzed and the ultimate deformation of the rest merely added on. For time scales of duration comparable to both  $\tau_M$  and  $\tau_K$ , but longer than  $\tau_1$  and  $\tau_2$ , only the Maxwell and the first Kelvin element need be considered and only the final response of  $c_1$  and  $c_2$  added. Further simplification may be achieved by suitably modifying the parameters of elements analyzed, so as to include the ultimate responses of the rest. Such a simplified model is shown in Figure 20.

Using this simplified model as a basis, relations analogous to elastic equations may be developed for cases discussed earlier, to which the elastic-viscoelastic analogy is applicable. For a homogeneous prismatic member, the following general differential equation covers bending, buckling, and vibration behavior:

$$\left[ \frac{I}{\alpha_M} \ddot{w}_{xxxx} + \left( \frac{1}{\tau_K} \right) \dot{w}_{xxxx} \right] = \left[ \frac{\partial^2}{\partial x^2} + \frac{B}{\tau_K} \frac{\partial}{\partial t} + \frac{\tau_K}{\alpha_M} \right] (p \pm Pw_{xx} + m\ddot{w})$$

where

$$B = 1 + \frac{\tau_K}{\tau_M} + \frac{\alpha_K}{\alpha_M}$$

$\alpha_M$  and  $\alpha_K$  = compliances of the Maxwell and Kelvin springs respectively

$\tau_K$  and  $\tau_M$  = the retardation and relaxation times of the Kelvin and Maxwell elements respectively  
 $I$  = moment of inertia of prismatic member  
 $w$  = deflection of the member  
 $\dot{w}$ ,  $w_x$  = derivatives of  $w$ , with respect to time and distance along the member respectively  
 $p$  = distributed transverse load (static bending)  
 $P$  = axial load (buckling)  
 $m$  = mass per unit length of the member (vibration).

The equation for any mode may be obtained by retaining the corresponding term from among  $p$ ,  $M$ , and  $m$  and setting the rest equal to zero. The actual solutions are involved and recourse may have to be taken to numerical and other approximate methods.

The elastic-viscoelastic analogy does not apply to reinforced or prestressed concrete as the stresses and strains are interdependent. In the case of reinforced or prestressed concrete members where a uniform strain distribution may be assumed, relatively simple expressions of the type developed by Ross,<sup>(40)</sup> and discussed earlier, could be derived. In other cases, however, the relationships become involved and approximate methods of estimating the strain and stress distribution appear to be the most convenient.

A rigorous analysis of the nonlinear model is, as pointed out earlier, prohibitively involved even for simple cases, and a step-by-step approach seems to be the only practical possibility.

Freudenthal and Ross have reported<sup>(105)</sup> experimentally determined values of the linear and nonlinear coefficients of the model shown in Figure 19 for concrete of various compositions. Figures 21 and 22 show the effect of

time on stress distribution in a reinforced concrete section under flexure and an axially prestressed section, estimated on the basis of the aforesaid parameters, using a step-by-step procedure. Both linear and nonlinear behaviors have been considered.

c. Torroja and Paez<sup>(66)</sup> have proposed a model using elastic, viscous, and friction elements (Figure 23-b) to simulate the inelastic behavior of concrete and have used it as a basis for the analysis of plain and reinforced concrete sections. As the model response is nonlinear in relation to the stress, tracing the behavior step by step and fiber by fiber through time is the only practicable approach for solving the general case. Figure 24 shows the redistribution of stresses with time for a reinforced concrete section under flexure at simulated service and ultimate load levels.

A simplified alternative approach makes use of an invariable relative stress strain curve (Figure 25) and the ultimate strength and associated strains corresponding to failure occurring after various durations of sustained load. The stress and strain distribution for a given duration of sustained load is estimated as follows:

a. A trial strain distribution (subject to compatibility requirements) is assumed across the section.

b. The strains are divided by the ultimate strain corresponding to the elapsed time under consideration, thus obtaining a relative strain distribution.

c. Using the relative stress strain curve, relative stresses are computed at a sufficient number of locations across the section and converted to actual stresses by multiplying with the strength corresponding to the

elapsed time.

d. Stresses in the reinforcement are computed from the actual strains assumed in step a.

e. The resulting internal forces are checked for equilibrium against the external load.

f. Steps a through b are repeated with different trial strain distributions until equilibrium is attained.

The assumption of an invariable relative stress-strain curve implicitly disregards the effect of the previous stress history of a fiber on its static strain at a given time and vice versa. It thus presumes a unique relationship between relative strain and relative stress which is not necessarily the case. For example, in a reinforced concrete section under flexure, which has been under load for some time, certain fibers may be strained but not stressed and vice versa, or in extreme situations a fiber under tensile strain may even be carrying compressive stress and vice versa. Despite these limitations, the procedure outlined gives results which check closely enough for practical purposes with solutions obtained by more elaborate methods, in which the stress-strain history is traced fiber by fiber.

The relative stress-strain curve may be considered as a refinement of the effective modulus method, extending it to nonlinear stress-strain relations as would arise in analyzing a section under a sustained load approaching the ultimate.

d. Using a Burghers body as a rheological model for concrete, Hansen<sup>(80)</sup> has developed an approximate procedure for estimating loss of prestress due to creep of concrete.

#### IV. CREEP STRAIN

##### A. GENERAL NATURE AND MAGNITUDE

In the study of the rheological behavior of concrete, the following deformational characteristics are often considered:

1. Curve of total deformation against time for a given stress history, usually a given constant stress.
2. Shrinkage curve, i.e., curve of deformation against time, for zero stress.
3. Stress-strain curves for different stress or strain rates.
4. Curve of instantaneous deformation against time.
5. Creep-time curve obtained by deducting from the total deformation curve the corresponding ordinates of the shrinkage curve and either the initial ordinate of the instantaneous deformation curve for all times, or the corresponding ordinates of the instantaneous deformation curve.
6. Curve of stress relaxation for constant deformation.
7. Deformational response to programmed sustained stresses.

If a concrete specimen is subjected to a constant uniform stress and the corresponding deformations are measured over a considerable length of time and the deformations are plotted against time, a typical pattern will be observed for all such curves. As soon as the stress is applied considerable instantaneous deformation is observed. The total observed

deformation continues to increase, of course at a decreasing rate. The first part may be called instantaneous deformation while the increase of deformation above this may be called time-dependent deformation.

If the stress is removed immediately after it is applied much of the instantaneous deformation disappears and corresponds to what is called instantaneous elastic deformation. The part of the initial instantaneous deformation that does not vanish is called initial set or instantaneous plastic deformation.

If, after any duration of sustained stress on the specimen, the stress is removed, an instantaneous recovery takes place which is practically the same in magnitude as the initial instantaneous deformation observed on the application of load, the difference between the two being attributable either to the increase in the modulus of elasticity with age or possible time-dependent plastic set. The deformation corresponding to this part is known as instantaneous elastic recovery. This is followed by a recovery of deformation which increases with time at a decreasing rate and becomes practically constant after a sufficient period has elapsed. This part of the recovery of deformation is termed the time-dependent recovery, the phenomenon itself being called delayed recovery, or primary creep.

The residual deformation retained after the recovery is complete includes deformations



due to shrinkage and permanent set. If these are accounted for, the remaining deformation is termed the permanent creep. This is sometimes also called secondary creep, in contrast with the deformation due to delayed recovery, which is called primary creep. These deformations are indicated in Figure 26.

The ultimate creep deformations per unit stress range from  $2 \times 10^{-7}$  to  $2 \times 10^{-6}$  inches per inch; an average figure is about  $10^{-6}$  inches per inch, which is about three times the elastic deformation of concrete with a modulus of elasticity of  $3 \times 10^6$  psi. Approximately one-fourth of the ultimate creep occurs within the first month or so and about one-half occurs within a year (Figure 10). These figures give a rough idea of the size and shape of a typical curve for creep in concrete.

A creep test is usually designed to give a creep-time curve for some reasonable period of time. The creep-time curve thus obtained is then utilized for studying the long time behavior of concrete structures, e.g., deflections, stress redistribution, loss of prestress, etc. As the stresses applied in these cases are not necessarily constant, special methods have to be used for estimation of these quantities from a given experimental creep-time curve obtained under constant applied load. One of the suggested quantitative formulas for estimating creep under constant stress may be used instead of an actual creep-time curve, provided the required coefficients for the particular type of concrete under the particular environments are available.

## B. EXPERIMENTAL DETERMINATION

Experimental techniques for the study of creep in concrete are still far from being standardized. Individual investigators have used a variety of equipment and methods, some of them devised for a specific purpose. Some of the typical techniques are briefly described

and any special advantages or shortcomings pointed out. In addition, some of the general problems which confront the research worker in this field, and to which fully satisfactory solutions are not yet available, are discussed.

### 1. Test Specimens and Loading Devices

a. Uniaxial compression. By far, the largest number of creep tests on concrete have been performed under uniaxial compression. The specimens have invariably been prisms or cylinders of various sizes and dimensional ratios.

The most commonly used loading devices for obtaining uniaxial compression are briefly described and their general characteristics discussed.

Spring loaded systems: These systems are essentially of one or more heavy coil springs bearing on the specimen and held in a compressed position against a suitable frame. The device has the advantage of simplicity. With spring compliance considerably higher than that of the specimen, the magnitude of the sustained load is not affected to any significant extent by the change in length of the specimen due to creep.

For larger specimens and consequently heavier loads, the system tends to become unwieldy. In addition, checking and adjustment of the load with this type of setup can be rather inconvenient.

A typical spring loaded system is shown in Figure 27.

Simple hydraulic systems: Hydraulic loading systems are flexible and compact and are capable of applying heavy loads. The auxiliary equipment is relatively light and can serve units singly or jointly, as desired. Application and measurement of load is simple and reliable. A very simple and self-contained setup, used successfully by L'Hermite,<sup>(74, 108)</sup> is shown in Figure 28.



The maintenance of a sustained load in a simple hydraulic system, however, presents certain practical difficulties. The applied load is quite sensitive to small changes in the length of the specimen and also to small unavoidable leakage of hydraulic fluid. Such systems therefore need frequent adjustment.

**Stabilized hydraulic systems:** The advantages of a spring loaded system, without its drawbacks, can be easily incorporated in a hydraulic system to obtain sustained loads. An auxiliary spring loaded hydraulic cylinder included in the hydraulic loading system provides the necessary stability of pressure against small volume changes. Such stabilizers may either be used individually or with several interconnected loading units.

Figure 29 shows such a stabilized system used at the University of Illinois.<sup>(123)</sup>

b. **Biaxial compression.** Ross<sup>(67)</sup> described the use of short square prisms (6 by 6 by 2-in.) for the study of biaxial compression. Spring loads on the narrow faces provided the required stress field (Figure 30). Special care was necessary to insure uniform load application. Rubber packing was used to avoid stress concentrations at contact surfaces.

c. **Triaxial compression.** Very little information is available on creep under triaxial compression. Davis, Davis, and Hamilton<sup>(25)</sup> describe a limited number of such tests using a hydraulic cell surrounding the specimen, which was sealed in a copper jacket. The lateral pressures were adjusted continually to maintain the lateral dimension of the specimen constant, while axial strains were measured under sustained axial loads.

d. **Uniaxial tension.** The study of creep of concrete under tensile stresses, presents several experimental problems, the most important being the application to the specimen of a uniformly distributed tensile stress.

Direct application of traction at the ends of prismatic specimens has been used by McMillan<sup>(108)</sup> and others, by using anchorages embedded along the axes at the specimen ends to which the necessary tensile loads were applied by hydraulic or other means. It appears, however, extremely difficult to eliminate all eccentricity and stress concentrations in such an arrangement.

Ross<sup>(67)</sup> describes an ingenious arrangement, in which a specimen in the form of a thin walled hollow cylinder with open ends is used. A flexible pressure bag is used to apply uniform radial compression on the inner face resulting in circumferential tension with an approximately uniform distribution across the wall thickness. The ends of the pressure bag are supported independently, thus relieving any axial stress in the cylinder (Figure 31). Axial compression may also be applied simultaneously, which would produce a tension-compression biaxial stress field.

e. **Flexure.** Several investigators<sup>(33, 71, 123, 124)</sup> have used prismatic beams under lateral loads for the study of creep strains in the compression and tension zones. As relatively lighter loads are required, dead weights or dead weight and lever arrangements (Figure 32) have been successfully used which greatly simplify measurement and maintenance of the test loads.

f. **Torsion.** Creep tests under torsion have been reported by Anderson,<sup>(104)</sup> Duke and Davis,<sup>(41)</sup> and Glucklich and Ishai.<sup>(111)</sup> The latter used cylindrical specimens loaded under torsion by a dead weight and pulley arrangement (Figure 33). As in flexure, the loads required are relatively small and easily applied by dead weights or dead weight and lever arrangements.

## 2. Measurement of Strains

Creep strains at specimen surfaces have

been measured mostly by means of mechanical or mechanical-optical gages, because of their independence from time effects, unlike the electrical resistance gage, in which the creep of the bonding cement can be a serious problem.

Demountable gages using a dial indicator movement, typified by the Whittemore gage, have been used by a majority of investigators. A sensitivity of  $10^{-5}$  in./in. is easily obtained with a gage length of 8 to 10 in.

A very simple single lever gage, for permanent installation, has been described by Ross<sup>(67)</sup> (Figure 34) in which a mechanical magnification is supplemented by the use of a microscope. A sensitivity of better than  $2 \times 10^{-6}$  in./in. has been obtained over a 2-in. gage length. The instrument is very light and easily installed.

Use of electrical resistance gages has been reported by some investigators. Davis, Davis, and Hamilton<sup>(25)</sup> used embedded gages for measuring internal strains in specimens. Carbon-pile telemeters were used by Jensen and Richart.<sup>(34)</sup> Duke and Davis<sup>(41)</sup> used Carlson type strain meters for surface and internal applications in their triaxial tests. Kesler, Chang, and Vaishnav<sup>(70, 123)</sup> used electrical resistance gages for limited observations of creep strains in compression. The long time stability and satisfactory performance of electrical resistance gages for measurement of creep strains in concrete is, however, not fully established.

A special electrical resistance strain gage using an unbonded element has recently been developed,\* which has a built-in standardizing device. The instrument is mounted permanently and the zero reading can be checked at any time, thus eliminating the problem of

instability due to creep of the gage mounting during prolonged measurements.

Any instrumentation for measurement of creep strains (except demountable gages) must of necessity be exposed to conditions of sustained humidity, which imposes stringent requirements on long term stability in such environment. This poses special problems in the development of suitable equipment.

Measurement of creep strains other than at the surface of a concrete specimen is still a serious problem.

### 3. Control of the Environment

As is well known, the temperature and humidity conditions, both inside the specimen and outside, have profound influence on the time-dependent deformation of concrete. The problem of controlling and measuring the pertinent parameters, although fully recognized, has not been approached with the rigor which its importance warrants. Investigators seldom mention the details of the equipment employed for this purpose, especially the adequacy of control actually attained.

Conventional air conditioning equipment can ensure control of only the gross environment, whereas it is the environment immediately surrounding the specimen that is of primary importance.<sup>(108)</sup> Much closer control of the local environment is thus indicated if a basic study of the problem is to be attempted, or an effective comparison of creep data from different sources is desired.

The other aspect of the problem is associated with the measurement of the thermal and hygrometric parameters inside the specimen. Since the time required for attaining a given degree of hygrometric and thermal equilibrium increases rapidly with specimen size,<sup>(79)</sup> except for the smallest sizes, most specimens of conventional sizes cannot be expected to attain any considerable degree of equilibrium

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\* Monfore standardizing strain gage, U. S. Pat. No. 2,889,528.

during the first few months. This lack of equilibrium has been shown<sup>(79, 37, 38)</sup> to have very significant influence on the magnitude of the observed creep strains. Any realistic interpretation of experimental data on creep must therefore consider this factor. Measurement of temperature gradients inside a specimen does not pose any special problems. A satisfactory solution to the problem of measuring moisture gradients, especially inside a laboratory specimen, is still awaited.

### C. EXPRESSIONS FOR CREEP STRAIN

Creep deformation, by definition, is a function of stress and time. Creep deformations are arrived at, in practice, by subtracting from the total observed deformations the sum of the deformations corresponding to the instantaneous effect of stress and deformations due to the time-dependent effect under a zero stress condition. Thus an expression for creep deformations should represent creep as a function of stress and time. Several authors have presented expressions for creep deformations but most of them, except Straub,<sup>(22)</sup> have assumed creep deformations in linear proportion to the applied stress. Creep deformations are usually expressed in terms of specific or unit creep, i.e., creep per unit stress, as a function of time. The question of the validity of assuming creep as directly proportional to the first power of stress has been discussed earlier.

At least thirteen different authors have proposed formulas for creep. These are briefly reviewed below:

Straub<sup>(22)</sup> presented an exponential function of the type  $\epsilon_c = K\sigma^p t^q$  where

- $\epsilon_c$  = creep strain
- $K$  = an empirical coefficient
- $t$  = time after application of stress
- $\sigma$  = applied sustained stress

$p, q$  = empirical exponents.

By not specifying  $p$  to be unity, this form does not imply necessary creep-stress proportionality.

Shank<sup>(29, 30)</sup> incorporated the idea of creep-stress proportionality and expressed specific creep as  $c = C^a \sqrt{t}$ ;  $C$  and  $a$  being empirical constants. He analyzed creep data from more than one hundred tests, mainly those of Davis and Glanville and those from Ohio State University, and derived values of  $C$  and  $a$ . He recognized that the observed data did not fit the equation well except for the periods between one day to one year. He obtained values for different cements, loading ages, storage conditions and aggregate types. He also suggested how the calculated values might be modified in case of periods greater than one year. He found the average values for  $C$  and  $a$  for the data he analyzed to be 0.13 and 3 respectively, with chances of 25 to 1 that all possible variations would be within 61 per cent for  $C$  and 39 per cent for  $a$ .

Ross<sup>(31)</sup> proposed a hyperbolic expression of the type  $c = \frac{t}{a + bt}$ , where  $c$  = unit creep,  $t$  = time in days after loading, and  $a$  and  $b$  are experimentally determined constants for various conditions.

Lorman's<sup>(36)</sup> formula,  $\epsilon_c = \frac{mt}{n + t} \sigma$ , is essentially the same as that of Ross's mentioned above, except that  $\epsilon_c$  represents the creep strain for a stress  $\sigma$  sustained for time  $t$ ;  $m$  is the creep coefficient and is equal to the ultimate creep strain per unit of stress;  $n$  represents the constant time at which half the value of the ultimate creep is attained.

Thomas<sup>(24)</sup> developed this exponential formula for creep:

$$c = c_{\infty} \left[ 1 - e^{-A(t+a)^b} - a^b \right]$$

where  $c$  = specific creep at time  $t$ ,  $c_{\infty}$  =

limiting specific creep, and  $A$ ,  $a$ , and  $b$  are empirical constants.

McHenry<sup>(39)</sup> presented the following exponential function for creep composed of two parts, the latter part representing the effect of the age at loading:

$$c = A(1 - e^{-rt}) + Be^{-pk} (1 - e^{-qt})$$

where  $c$  = specific creep,  $k$  = the age at the time of loading,  $t$  = time after loading, and  $A$ ,  $B$ ,  $p$ ,  $q$ , and  $r$  are empirical constants.

This equation may be simplified to  $c = (A + Be^{-pk}) (1 - e^{-rt})$ , assuming the exponents  $r$  and  $q$  to be equal. The equation is based on the postulate that when a constant sustained load is applied to concrete of stable structure and properties, a certain amount of potential creep is introduced. The rate of creep at any time is proportional to the amount of potential creep remaining; and that there also exists an exponentially disappearing transient creep property representing the effect of aging superposed on the stable property. The equation can be used to construct a three-dimensional creep surface with the axes representing specific creep strain, loading age, and the time after loading (Figure 15). The necessary five constants can be evaluated from laboratory creep tests for different ages at loading. The equation is claimed to be applicable to age at loading greater than five days and less than one year.

Glanville and Thomas<sup>(35)</sup> later proposed the following equation:

$$c = c_{\infty} [1 - e^{-f(t)}]$$

where  $c$  is specific creep at time  $t$ ,  $c_{\infty}$  is specific limiting creep and  $f(t)$  is an empirical time function.

Dischinger's<sup>(51)</sup> equation of creep is the same as the first part of McHenry's equation, i.e.,

$$c = c_{\infty} (1 - e^{-at})$$

where  $a$  is an empirical constant.

Koerner<sup>(64)</sup> applied Maxwell's relaxation theory to creep of concrete. He proposed the equation:

$$\epsilon_e + \epsilon_c = \sigma e^{t/\tau}$$

where  $\tau$  is the relaxation time (at the end of which the stress relaxes, at a constant deformation, to  $1/e$  times its initial value), and  $\epsilon_e$  is the instantaneous strain under the applied stress.

Friedrich<sup>(53)</sup> gave the following expression for creep for periods less than about four years:

$$c = C \sqrt[4]{\frac{t}{1400}}$$

where  $t$  = time in days. He assumed that creep virtually stops by the end of about 1400 days and that the values for creep thereafter should be taken as equal to  $C$ , i.e., creep at 1400 days.

Aroutiounian<sup>(125)</sup> proposed the following equation for creep:

$$c(T, k) = \phi(T) [1 - e^{-a(T-k)}]$$

where  $c(T, k)$  = specific creep of concrete at time  $T$  after casting for concrete loaded at age  $k$ ;  $\phi(T)$  = empirical time function; and  $a$  = empirical exponent.

Inge Lyse<sup>(110)</sup> proposed the following relation:

$$c(t) = S(1 - e^{-st})^p \frac{\sigma_a}{\sigma_{sh}}$$

where  $c(t)$  = specific creep at a time  $t$  after loading,  $\sigma_a$  = applied sustained stress on the concrete,  $\sigma_{sh}$  = magnitude of an equivalent sustained stress which would give creep equal to the shrinkage of the concrete at the given relative humidity,  $p$  = percentage of

cement paste in the concrete,  $s$  = rate of shrinkage, and  $S$  = limiting shrinkage per one per cent of paste in the concrete at the given relative humidity.

The following equation is used by the United States Bureau of Reclamation for expressing creep as a function of time:

$$c = f(K) \log_e (t + 1)$$

where

$\epsilon_c$  = specific creep strain at time  $t$   
in days after application of sustained stress

$f(K)$  = a function representing the rate of creep deformation with time.

The parameter  $f(K)$  has to be determined for each specific case. The equation is based on the assumption that the creep rate is inversely proportional to the time. According to this equation, creep increases indefinitely with time, although at a decreasing rate.

L'Hermite<sup>(74)</sup> gives the following exponential formula:

$$c = c_{\infty} \left[ 1 - e^{-(a_1 \log \frac{k+t}{k} + b t)} \right]$$

where  $c$  = specific creep,  $c_{\infty}$  = limiting specific creep,  $k$  = age at loading,  $t$  = creep time, and  $a$  and  $b$  are empirical constants.

The formula is based on the assumption that the fluidity (the reciprocal of viscosity) of concrete decreases as the age of the concrete increases.

Erzen<sup>(126)</sup> proposed the formula

$$c = A e^a \left[ 1 - (k/t)^b \right]$$

where  $c$  = specific creep strain;  $t$  = creep time;  $k$  = age at loading;  $A$ ,  $a$ , and  $b$  are empirical constants.

Expressions for creep, based on analogous rheological models, have been proposed by several authors, notably Ross,<sup>(40)</sup> Flügge,<sup>(55)</sup> Cowan,<sup>(127)</sup> Freudenthal and Roll,<sup>(105)</sup> Torroja and Paez,<sup>(66)</sup> Chang and Kesler,<sup>(70)</sup> and Glucklich and Ishai.<sup>(111)</sup> The expressions are discussed under the review of rheological models for concrete.

Figure 35 shows a comparison of some of the creep relations described above. Comparable magnitudes of the predicted ultimate creep have been used for the curves.

## V. INTERNAL STRUCTURE OF CONCRETE

### A. GENERAL CONSIDERATIONS

The strength and deformation characteristics of concrete are, as would be expected, intimately related to the nature of its internal structure. A knowledge of the physico-chemical properties of the various ingredients and their structure at macroscopic, microscopic, and molecular levels is thus an essential step in investigating the mechanism governing the rheological behavior of concrete.

Brunauer<sup>(128)</sup> describes concrete as a three-component system, the three components being cement, water, and aggregate. Air may be considered as the fourth component. The relative proportions of these components and their interaction make concrete what it is.

The cement paste constitutes the active part of the concrete. It surrounds the aggregate and binds it together. The strength, creep, shrinkage, and other mechanical characteristics of concrete appear to be governed to a major extent by the quantity and quality of the cement paste matrix. The aggregate, on the other hand, plays a relatively passive, although not unimportant role, as a "moderator".

In view of its primary importance in determining the mechanical properties of concrete, the structure of the cement paste and the factors influencing it have received considerable attention.

### B. STRUCTURE OF THE CEMENT PASTE

At any stage of hydration, the structure

of the paste consists essentially of an agglomeration of unhydrated grains of cement which are interconnected by an aggregation of colloidal of microcrystalline particles,<sup>(66)</sup> i.e., the so-called cement gel. The products of hydration, although dispersed to a considerable extent in the liquid phase, form a more or less rigid framework. The controversy as to the colloidal or crystalline nature of these products is more apparent than real. In all probability the individual gel particles are colloidal in size and have at least a partially crystalline structure. The rigidity of the mass may then be attributed to the interacting forces between these dispersed particles; the intermicellar liquid may or may not be an essential factor.

Powers<sup>(129, 130)</sup> studied the structure of hardened cement paste using the water fixation and absorption techniques. On the basis of these studies he estimated the relevant physical properties of the cement gel. The nominal gel, defined as the actual gel and the crystalline calcium hydroxide associated with it, was found to have a density of about 2.15 g/cm<sup>3</sup>, a porosity of about 26 per cent, and a coefficient of permeability of about  $2 \times 10^{-5}$  cm/sec. The gel itself, corrected for the calcium hydroxide, was found to have an average density of 2.65 g/cm<sup>3</sup> and a specific surface of the order of 250 m<sup>2</sup>/g. The size of the gel particle, expressed as the diameter of an equivalent sphere, was estimated at approximately 100 Å.



The actual shape of the particles is of course not spherical, as will be discussed later. The mean size of the gel pores was estimated at 20 to 40 Å. The water content of saturated gel was found to be 3 to 4 times the volume of water corresponding to a monomolecular layer on the total surface of the gel.

Powers thus visualized the gel as a porous mass with a characteristic porosity and a very large specific surface, which gradually replaces the original water content of the paste, as hydration proceeds. The void system associated with the gel is referred to as the gel pores, while that portion of the originally water-filled space still unoccupied by the gel is termed the capillary pores.

On the basis of the above concept, he proposed a simplified model for the structure of hydrated cement paste (Figure 36) capable of simulating, at least qualitatively, the observed strength, deformation, volume change, and durability characteristics of concrete.

The water present in the cement paste can exist in several forms which may be classified as follows, depending on the manner in which it is held in the structure:

**Fixed water:** The water existing in chemical combination with the cement, as a constituent of the solid phase of the gel, is termed the non-evaporable or "fixed" water content of the paste.

**Free water:** The complement of the fixed water content is referred to as the "free" or evaporable water content. It can be further classified as capillary water, which is held by confinement alone, and gel water which is held by molecular bonds. The gel water in turn can either be held on the surface of the gel particles as absorbed water or held within their crystal lattice as zeolitic water.

Although the above classification appears well defined, the factual boundaries between

the various forms are known to exhibit considerable overlap. Exact experimental delineation of these boundaries is thus almost impossible.<sup>(131)</sup> As a consequence, certain arbitrary test conditions are employed for the determination of the fixed and free water contents at any stage of hydration. Powers<sup>(132)</sup> considered as fixed water that water which was retained by a sample dried to constant weight in vacuum at 23 C over magnesium perchlorate. Other criteria for differentiating between evaporable and non-evaporable water have been suggested. Powers and Brown-yard<sup>(129)</sup> define non-evaporable water as that part of the total water content that has a vapor pressure of not more than  $6 \times 10^{-4}$  mm mercury at 23 C. Copeland and Hayes<sup>(133)</sup> set the limit at  $5 \times 10^{-4}$  mm. The free water content and its fractions, occurring as gel water and capillary water, may then be estimated from the known original water content and characteristic porosity, composition, and specific gravity of the gel.

The above mentioned concept derives its conclusions about the nature of the gel indirectly from the phenomenological behavior of the hardened paste, i.e., the manner in which it takes up and holds moisture. A more direct evidence of the gel structure and its progressive development during hydration is provided by electron microscope studies.

Grudemo<sup>(134)</sup> obtained electron-micrographs showing the results of the hydration process. During the initial stage, a coarsely fibrous structure in the form of slender rods grows radially from cement grains into the surrounding solution. Where the cement particles are in contact, the rod-like formations link together at intersections through superposition of their atomic structure, probably providing the initial rigidity to the paste. Subsequently, the rod-like or partly fibrous



product gradually grows to fill the interstitial spaces. Simultaneously a steadily increasing number of bonds are formed at the contact areas, which probably constitute one of the sources of strength and rigidity of the paste. As the colloidal growth on the surface of the cement particles increases, the rate of hydration slows down. At this stage another component is generated which is composed of a felted mass of extremely thin wrinkled foils. In the second stage of hydration this component grows out and in the course of time fills up the meshes in the network formed by the rod-like initial structure. This mass finally becomes so strongly compressed that the individual components are no longer discernible. This corresponds to a third relatively stationary stage, wherein all changes proceed at a very slow rate.

The strength, as well as deformational characteristics, of cement paste and in turn of the concrete in all probability stems from the nature of the structure at the surfaces of these colloidal particles and their mutual bonds.

#### C. STRENGTH AS RELATED TO THE INTERNAL STRUCTURE

Studies by Hansen<sup>(135)</sup> indicated three distinct straight-line portions in the logarithmic plot of concrete strength versus age, corresponding to the three stages of hydration observed by Grudemo and described above. The first stage lasted for the first 30 hours, the second from 30 to 72 hours, and the last from 72 hours to 28 days.

Reinius<sup>(136)</sup> developed a theory for the short-time deformation and failure mechanism of concrete in terms of a mechanical model similar to the structure described later by Grudemo. The model was conceived as a lattice of elongated crystals of varying strength, connecting the unhydrated cement particles.

Through the analysis of an idealized structure, he was able to explain the observed modes of rupture and patterns of crack formation in concrete under various types of load, and also the deformational behavior up to rupture.

The Reinius theory is restricted to the explanation of the mechanism of strength and deformation. It does not include the influence of organismic and environmental factors on the associated mechanisms. The structural concept can, nevertheless, be extended to cover the above factors.

An increase in the water content of concrete increases the distances between the cement grains and consequently reduces the number of contact points and gel connections in a given volume. Since both strength and rigidity depend on the number of such contacts and links, the strength would decrease and short-time deformations would obviously increase with an increasing water-cement ratio, as observed.

The microstructural composition of the hydration products do not appear to depend to any substantial extent on the composition of the cement<sup>(134, 137, 130)</sup> or on the degree of hydration. It may therefore be expected that the strength and rigidity of concrete are not influenced significantly by the type and composition of the cement, except through the amount of gel present at a given age.

Powers and Brownyard<sup>(129)</sup> showed empirically that the strength of hardened cement paste is roughly proportional to the gel-space ratio  $X$ , i.e.,

$$X = \frac{\text{volume of gel}}{\text{original space available to gel}}$$

$$= \delta \frac{V_m}{w_o} = \delta \frac{kw_n(t)}{w_o}$$

where

$v_m$  = a constant proportional to the surface area of the gel

$w_o$  = weight of original water  
 (corrected for bleeding)  
 $w_m(t)$  = weight of non-evaporable water  
 at the age considered  
 $k$  = a constant depending on the  
 compound composition of the  
 cement  
 $\delta$  = a proportionality factor between  
 $V_m$  and the total volume of gel  
 $V_{hc}$ .

Powers,<sup>(130)</sup> in his later studies, proposed  
 a modified function  $X'$  for the gel-space ratio  
 which takes into account not only the volume  
 of the original water content, but also the  
 space vacated by the hydrating cement and  
 includes a correction for any original air  
 voids.

$$X' = \frac{\alpha V_{hc}}{\alpha V_c + V_o + V_a}$$

where:

$\alpha$  = degree of hydration at any age  
 $V_{hc}$  = gross volume of gel corresponding  
 to full hydration of the cement  
 content  
 $V_c$  = solid volume of the cement content  
 $V_o$  = volume of original water content  
 $V_a$  = volume of original air voids.

The strength  $f_c$  was then represented as a  
 function of the gel-space ratio as follows:

$$f_c = k (X)^p.$$

Empirical determinations indicated values of  
 $k$  around 30,000 and  $p$  about 3 for most  
 cements.

#### D. ELASTICITY AS RELATED TO THE INTERNAL STRUCTURE

Hansen<sup>(79)</sup> suggests that depending on the  
 relative rigidities of the hardened cement  
 paste and aggregate, the elastic behavior of  
 the aggregate-paste system may be considered  
 as that of a "combined hard" or "combined

soft" material.

A combined hard material is represented  
 by a continuous phase of high rigidity with a  
 low rigidity phase filling its voids. The  
 stress-strain behavior may then be analyzed  
 on the assumption of a parallel action of the  
 two components in carrying load, i.e., a  
 criterion of equal strain in both phases is  
 applicable. Thus the elastic modulus  $E$  of  
 the composite material is given by:

$$E = v_s E_s + v_h E_h$$

where:

$v_s$  = volume concentration of the soft  
 phase  
 $v_h$  = volume concentration of the hard  
 phase  
 $E_s$  = elastic modulus of the soft phase  
 $E_h$  = elastic modulus of the hard phase

A combined soft material, on the other  
 hand, is represented by a continuous low  
 rigidity phase and a dispersed high rigidity  
 phase. The components exhibit a series action.  
 A criterion of equal stress in both the phases  
 prevails. The elastic modulus of the composite  
 material is thus given by:

$$E = \frac{1}{\frac{v_s}{E_s} + \frac{v_h}{E_h}}.$$

In general, the rigidity of the contin-  
 uous phase in concrete (i.e., the cement  
 paste) is lower than that of the dispersed  
 phase (i.e., the aggregate). Concrete or mor-  
 tar may therefore be expected to behave as a  
 combined soft material. Hansen analyzed the  
 experimental data of Dantu<sup>(138)</sup> and La Rue<sup>(139)</sup>  
 and corroborated the above behavior. If, how-  
 ever, the aggregate has a lower rigidity com-  
 pared to the hardened paste, the behavior  
 appears to be represented more closely by a  
 combined hard material.

The cement paste itself may be considered

as composed of a combined soft material represented by the rigid particles of unhydrated cement particles embedded in a relatively soft matrix composed of the gel and capillary pores. The matrix in turn is a combined hard material with the gel forming the rigid skeleton and the capillary pores acting as the soft component. From the above consideration, Hansen (79) suggests the following expression for the elastic modulus  $E$  of concrete or mortar using relatively rigid aggregate particles:

$$\frac{1}{E} = \frac{v_1 v_3}{v_5 E_5} + \frac{v_1 v_4}{E_4} + \frac{v_2}{E_2}$$

where:

$v_1$  = volume concentration of cement paste in the paste-aggregate system

$v_2$  = volume concentration of aggregate in the paste-aggregate system

$v_3$  = volume concentration of gel and capillary pores in the paste

$v_4$  = volume concentration of unhydrated cement grains in the paste

$v_5$  = volume concentration of gel in the gel and capillary pore system

$v_6$  = volume concentration of capillary pores in the gel and capillary pore system

$E_2$  = elastic modulus of the aggregate

$E_4$  = elastic modulus of unhydrated cement grains

$E_5$  = elastic modulus of the gel.

#### E. RELATION BETWEEN ELASTICITY AND STRENGTH

As discussed above, both strength and elastic properties of the concrete may be estimated in terms of the volume concentration of its components and their relevant properties. In addition, the strength and rigidity of the gel may be considered, for all practical purposes, to be independent of the composition of the cement and degree of hydration.

The apparent correlation between the strength and elastic modulus of concrete, which

has so often been observed, could thus have a rational basis in the above concept.

#### F. PROBABLE MECHANISM OF SHRINKAGE

Cement paste may be considered as a limited swelling hydro-gel.<sup>(130)</sup> The necessary rigidity against unlimited swelling in contact with water is probably associated with the rod-like hydration products linking the rigid grains of unhydrated cement, while the swelling forces may be traced to the felted foil-like components filling the rigid network.<sup>(79)</sup> This felted mass possesses an enormous surface area with about  $7 \times 10^{16}$  gel particles per cubic centimeter. The interstitial spaces are therefore minute, being of the order of a few molecules of water in size. As a result of the close proximity and large area, van der Waal forces are prominent.

When a relatively dry mass of hardened paste is brought in contact with water, the water forces its way between the foils by virtue of the higher molecular attraction and forces them apart, resulting in an increase in gross volume or swelling. Abstraction of water results, similarly, in a reduction of volume or shrinkage. The absorbed or gel water is thus primarily responsible for volume change effects, while variations in the confined or capillary water have little influence on the volume. Due to the higher molecular attraction involved, the gel water is taken up first and given up last.

Volume change effects of the above type are not fully recoverable. Non-recoverable deformations could be caused by the formation of new bonds while the particles are closer together during shrinkage or some readjustment of the microstructure under the stresses created in it as it resists volume change.

The actual ingress or egress of water from or to the environment are not necessary for the above type of volume changes. A change

of temperature can disturb the equilibrium between the gel and capillary water contents, thus causing their redistribution. This results in the so-called hygrothermal volume change.<sup>(130)</sup> A partly saturated specimen with an internal relative humidity of about 70 per cent exhibits the effect to the maximum extent.

An alternative hypothesis for the pheno-

menon of shrinkage was proposed by Freyssinet,<sup>(57)</sup> according to which the progressively increasing miniscular curvature of the capillary water, caused by dessication, subjects the gel solids to compressive stresses, which in turn results in a volume reduction or shrinkage. Experimental evidence, however, appears to support the gel water concept of shrinkage rather than the capillary water hypothesis.<sup>(66)</sup>

# FIGURES





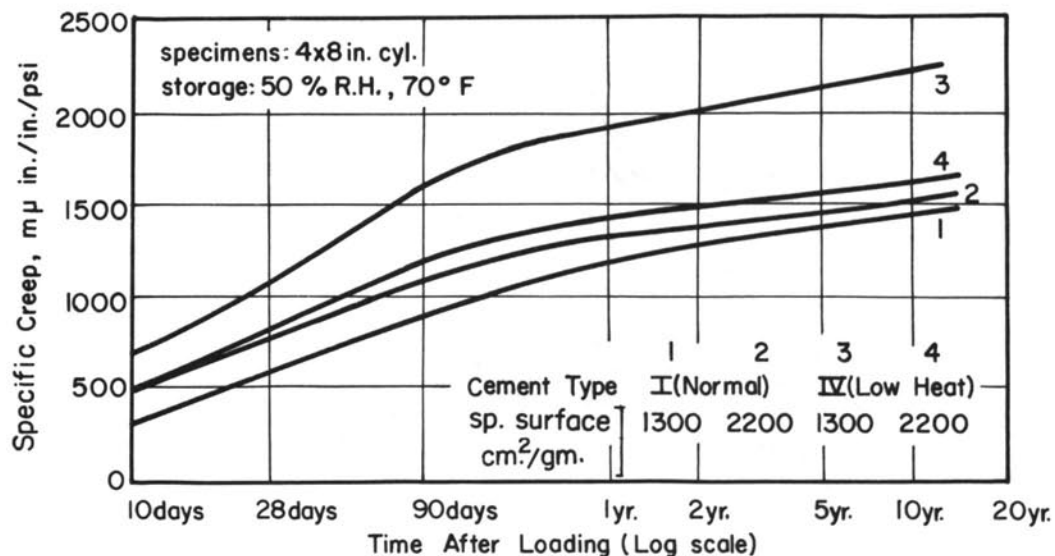


FIGURE 1. EFFECT OF COMPOSITION AND FINENESS OF CEMENT ON CREEP

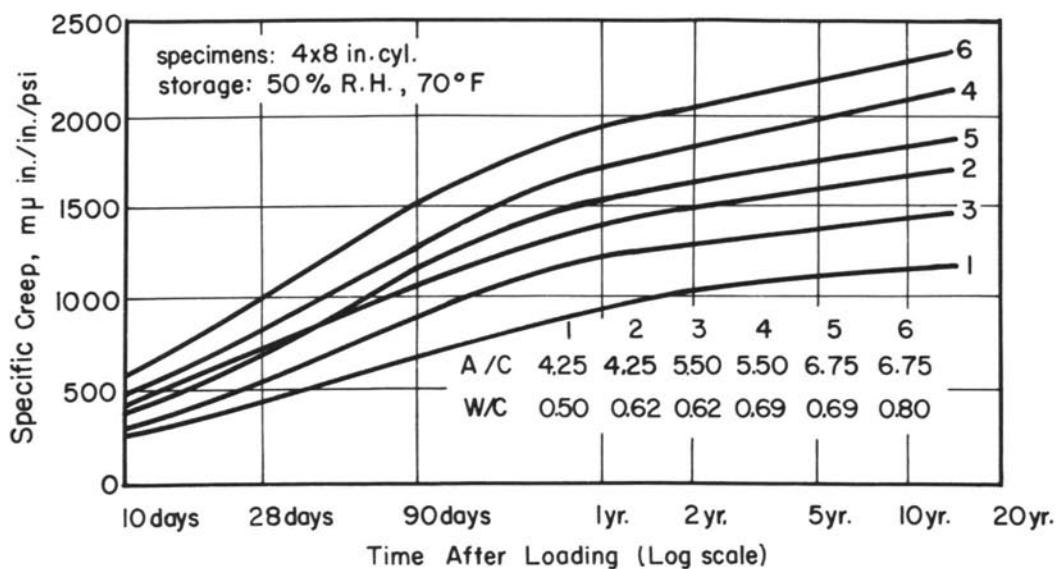


FIGURE 2. EFFECT OF AGGREGATE-CEMENT AND WATER-CEMENT RATIOS ON CREEP



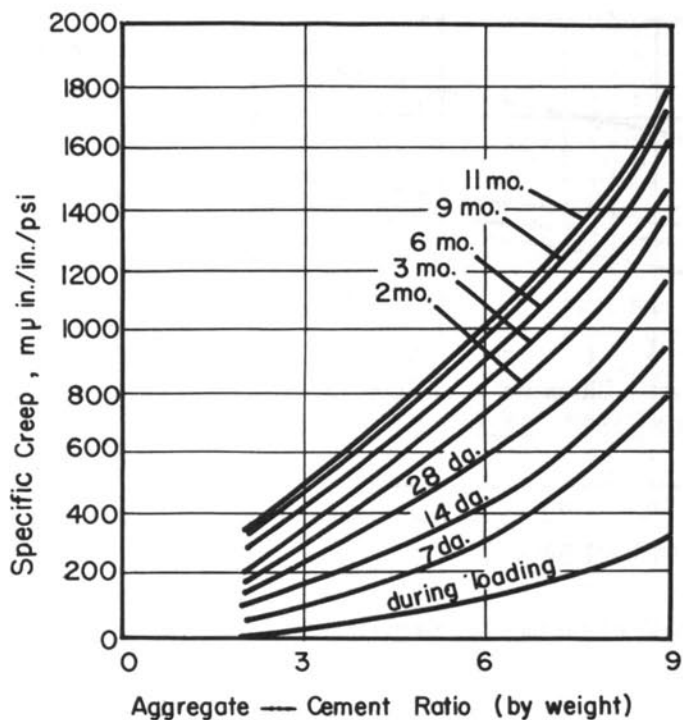


FIGURE 3. EFFECT OF CEMENT CONTENT ON CREEP

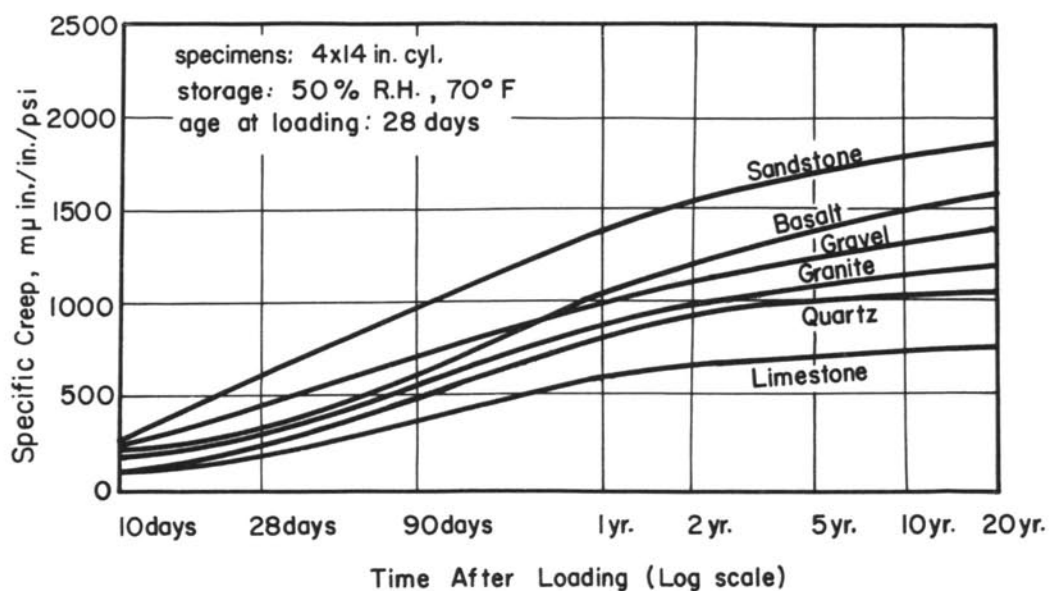


FIGURE 4. EFFECT OF TYPE OF AGGREGATE ON CREEP

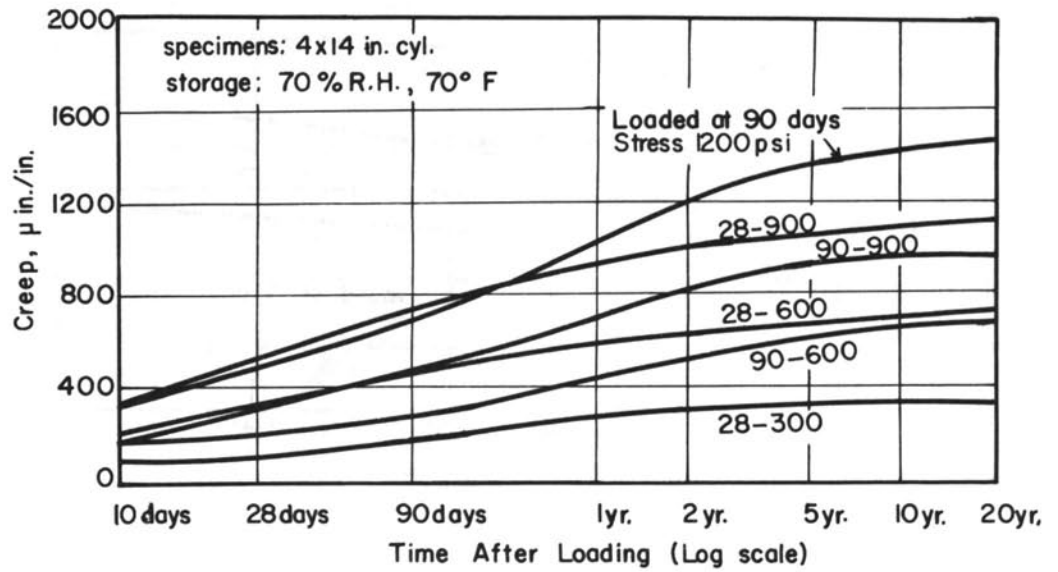


FIGURE 5. EFFECT OF AGE AT LOADING AND STRESS INTENSITY ON CREEP

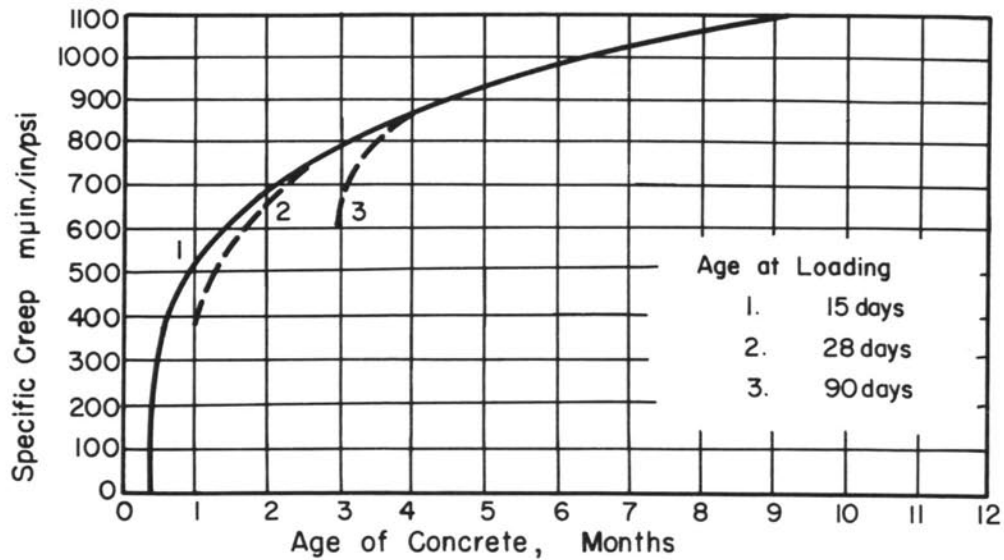


FIGURE 6. EFFECT OF AGE AT LOADING ON CREEP

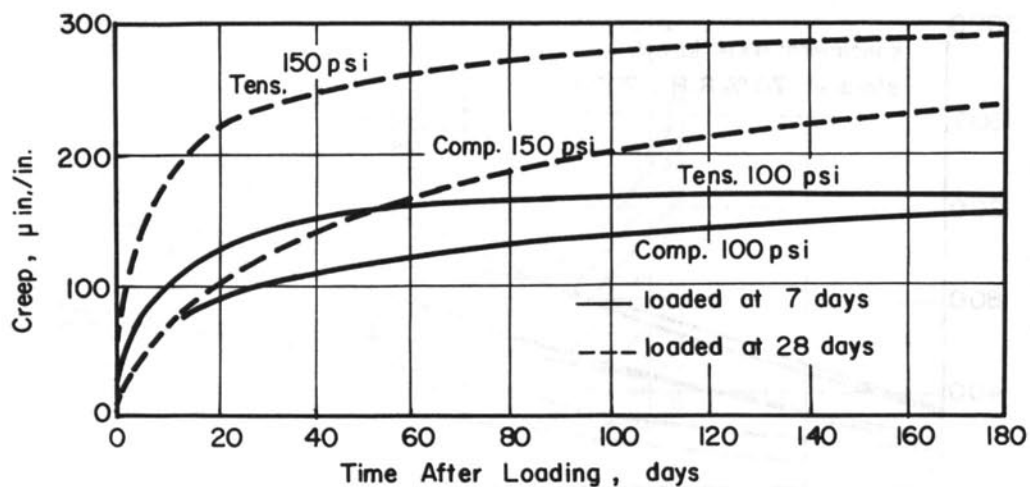


FIGURE 7. RELATIVE MAGNITUDE OF CREEP IN COMPRESSION AND TENSION

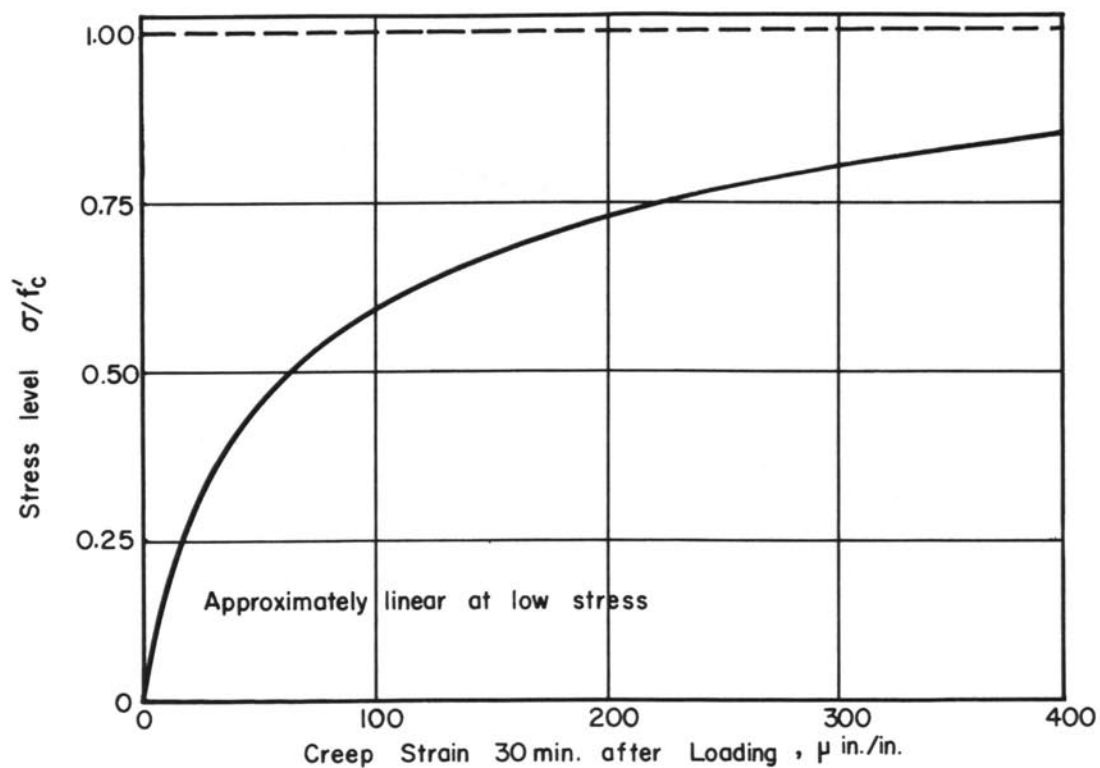


FIGURE 8. TYPICAL VARIATION OF CREEP STRAIN WITH STRESS LEVEL

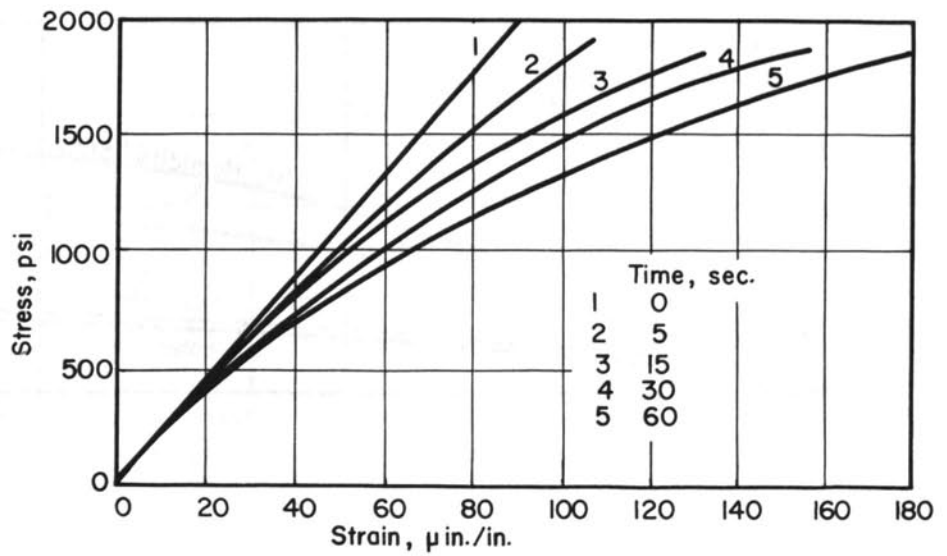


FIGURE 9. TYPICAL ISOCHRONIC STRESS-STRAIN RELATIONS FOR CONCRETE

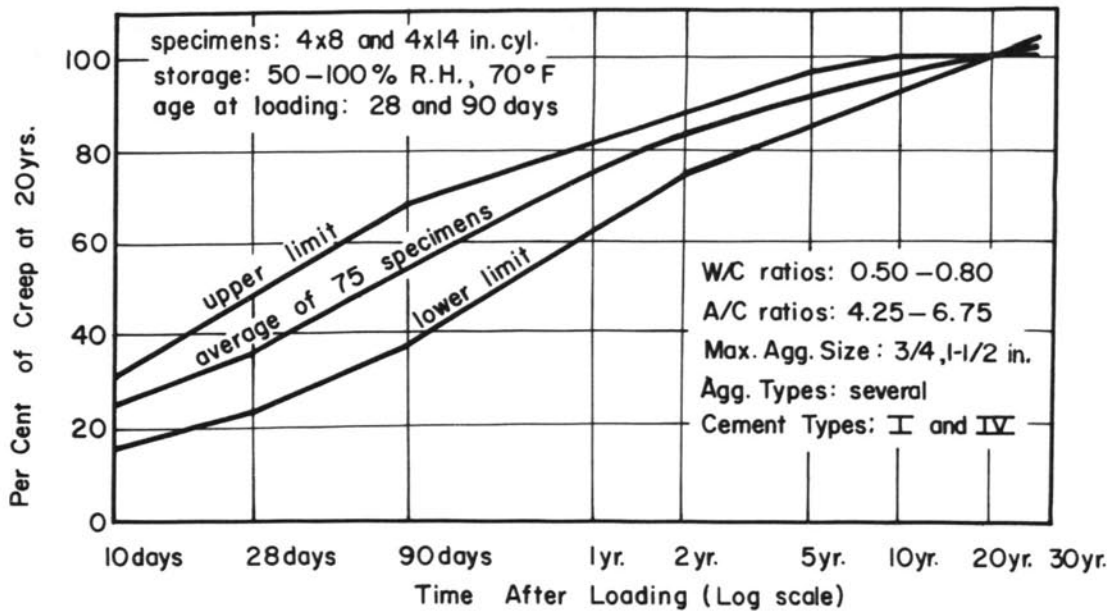


FIGURE 10. RELATIVE CREEP-TIME RELATIONSHIPS

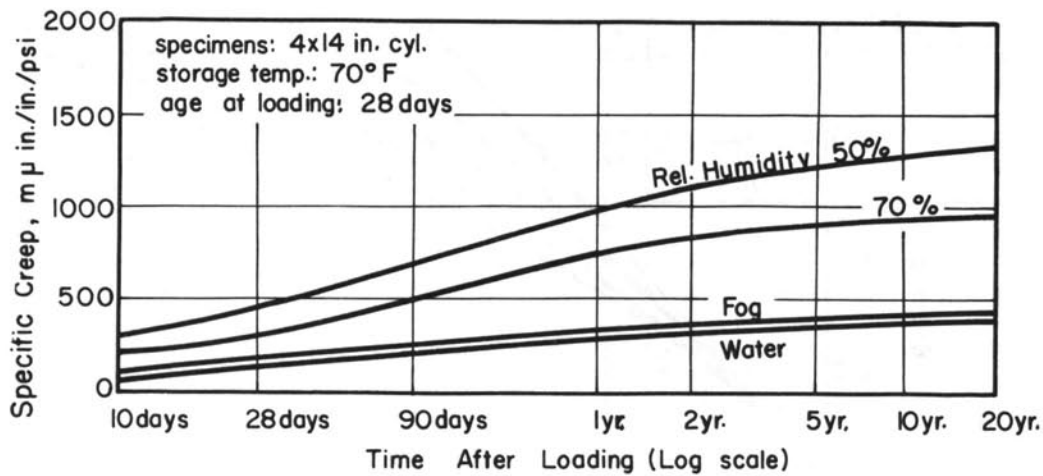


FIGURE 11. EFFECT OF MOISTURE CONDITION OF STORAGE ON CREEP

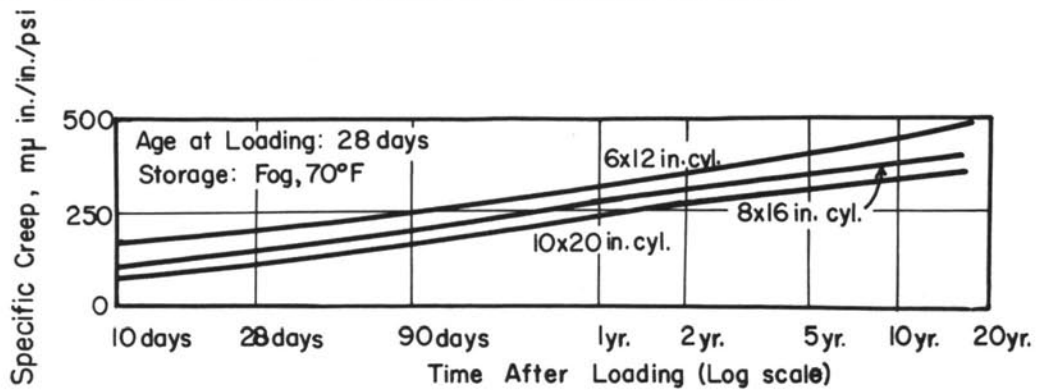


FIGURE 12. EFFECT OF SPECIMEN SIZE ON CREEP

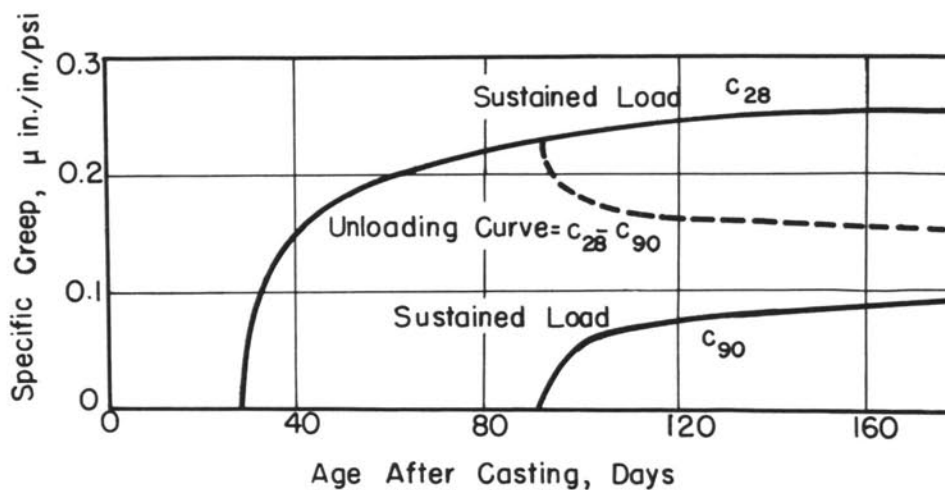


FIGURE 13. THE PRINCIPLE OF SUPERPOSITION FOR CREEP STRAINS

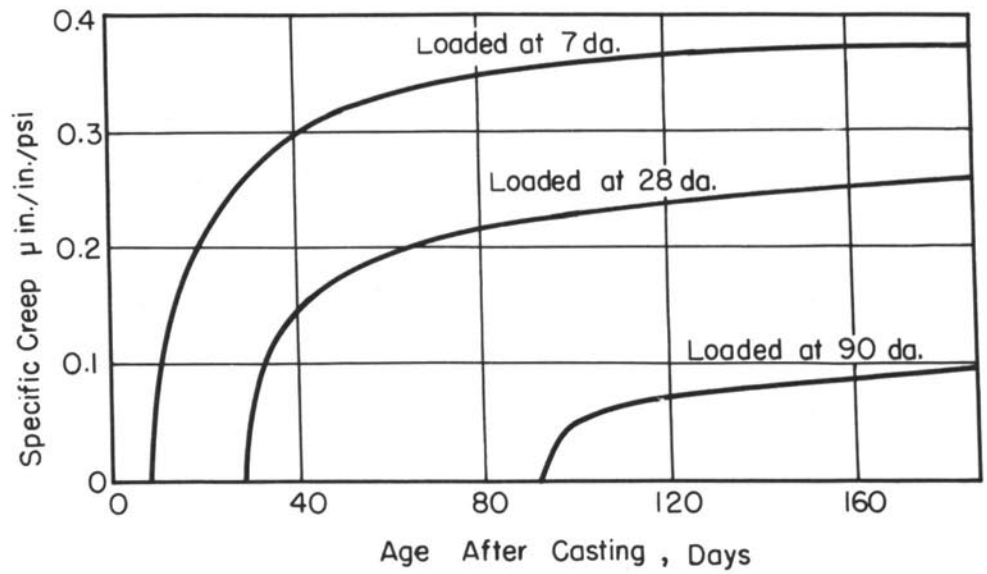


FIGURE 14. TYPICAL SPECIFIC CREEP CURVES FOR CONCRETE

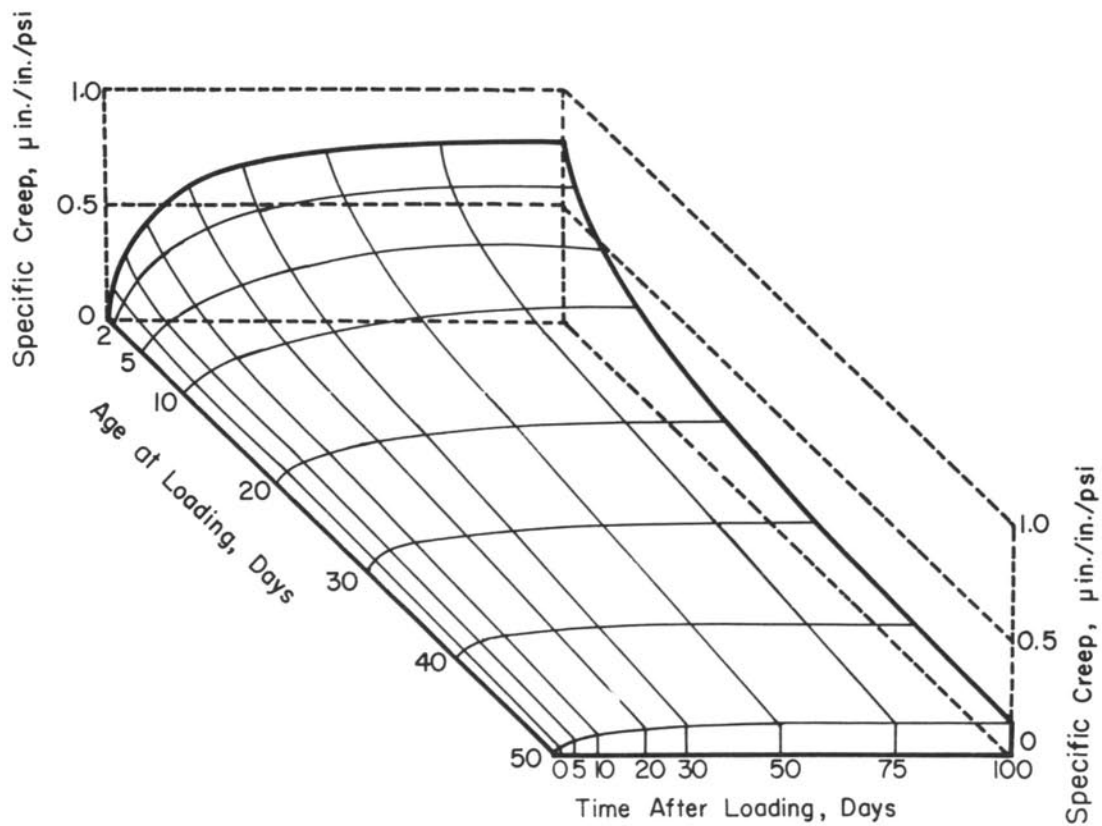


FIGURE 15. TYPICAL SPECIFIC CREEP SURFACE FOR CONCRETE

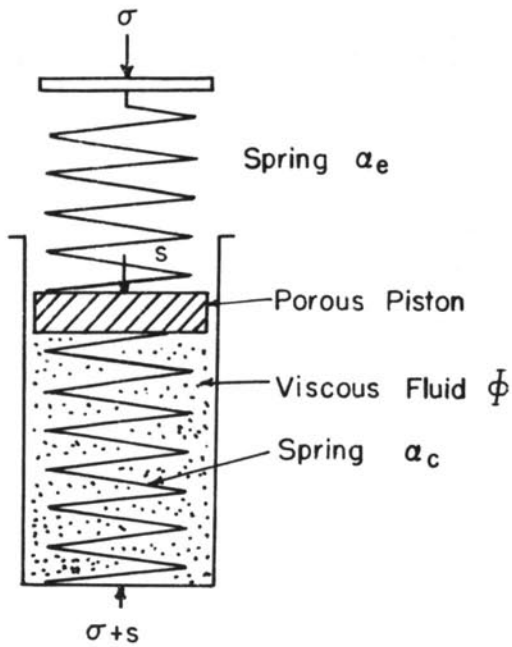


FIGURE 16. THREE-ELEMENT MODEL  
FOR CONCRETE AFTER ROSS

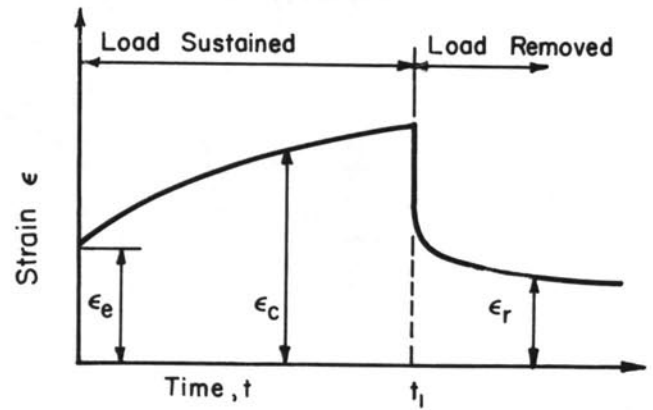


FIGURE 17. CREEP AND RECOVERY  
CURVE FOR ROSS MODEL

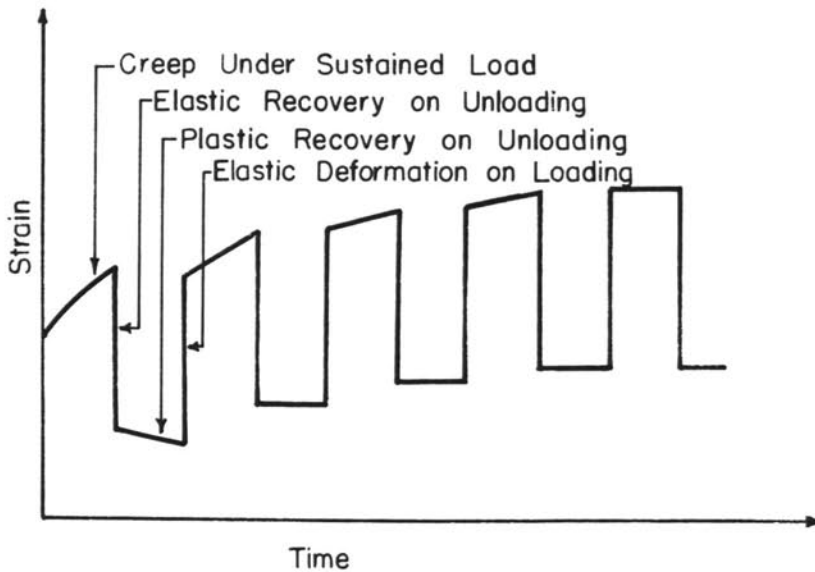


FIGURE 18. RESPONSE OF THE ROSS MODEL UNDER  
REPEATED LOAD-UNLOAD CYCLES

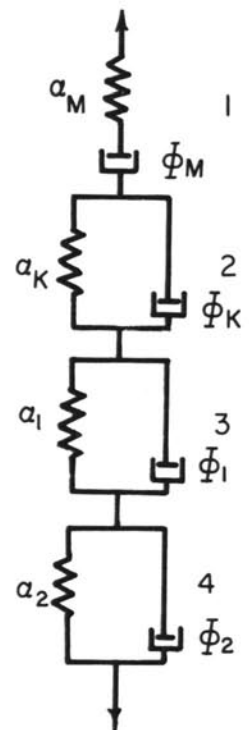


FIGURE 19. RHEOLOGICAL  
MODEL FOR CONCRETE  
AFTER FREUDENTHAL



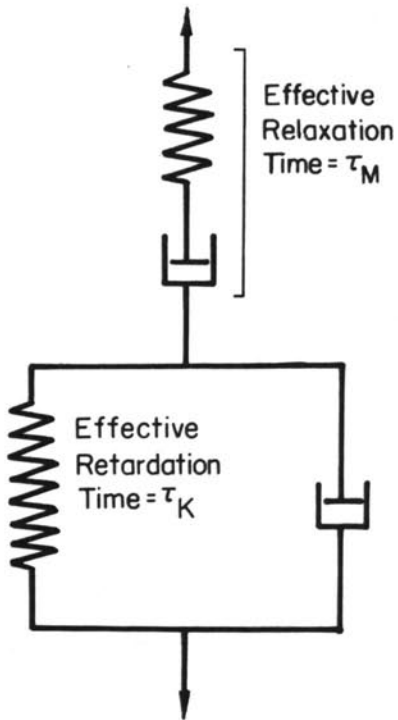


FIGURE 20. SIMPLIFIED MODEL FOR CONCRETE AFTER FREUDENTHAL

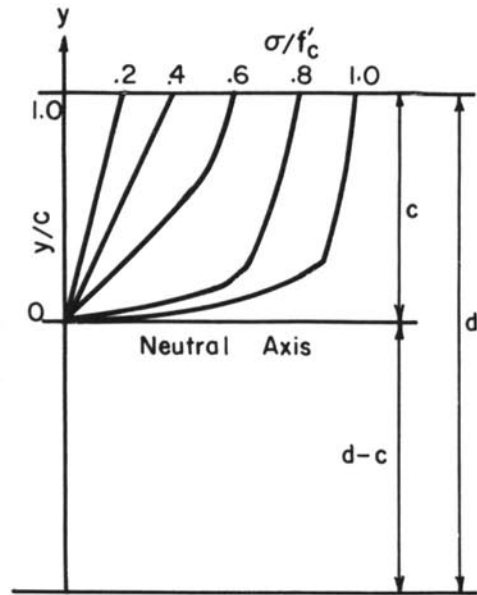


FIGURE 21. DISTRIBUTION OF COMPRESSIVE STRESSES ACROSS A RECTANGULAR SECTION UNDER FLEXURE (NEUTRAL AXIS CONSIDERED FIXED)

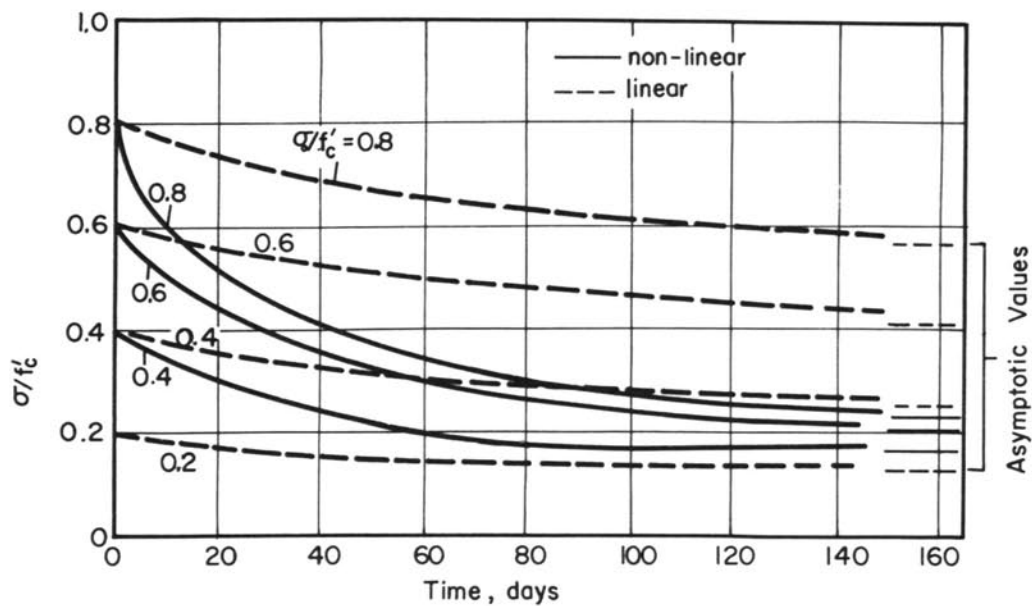


FIGURE 22. LOSS OF PRESTRESS FOR ASSUMED LINEAR AND NON-LINEAR BEHAVIOR

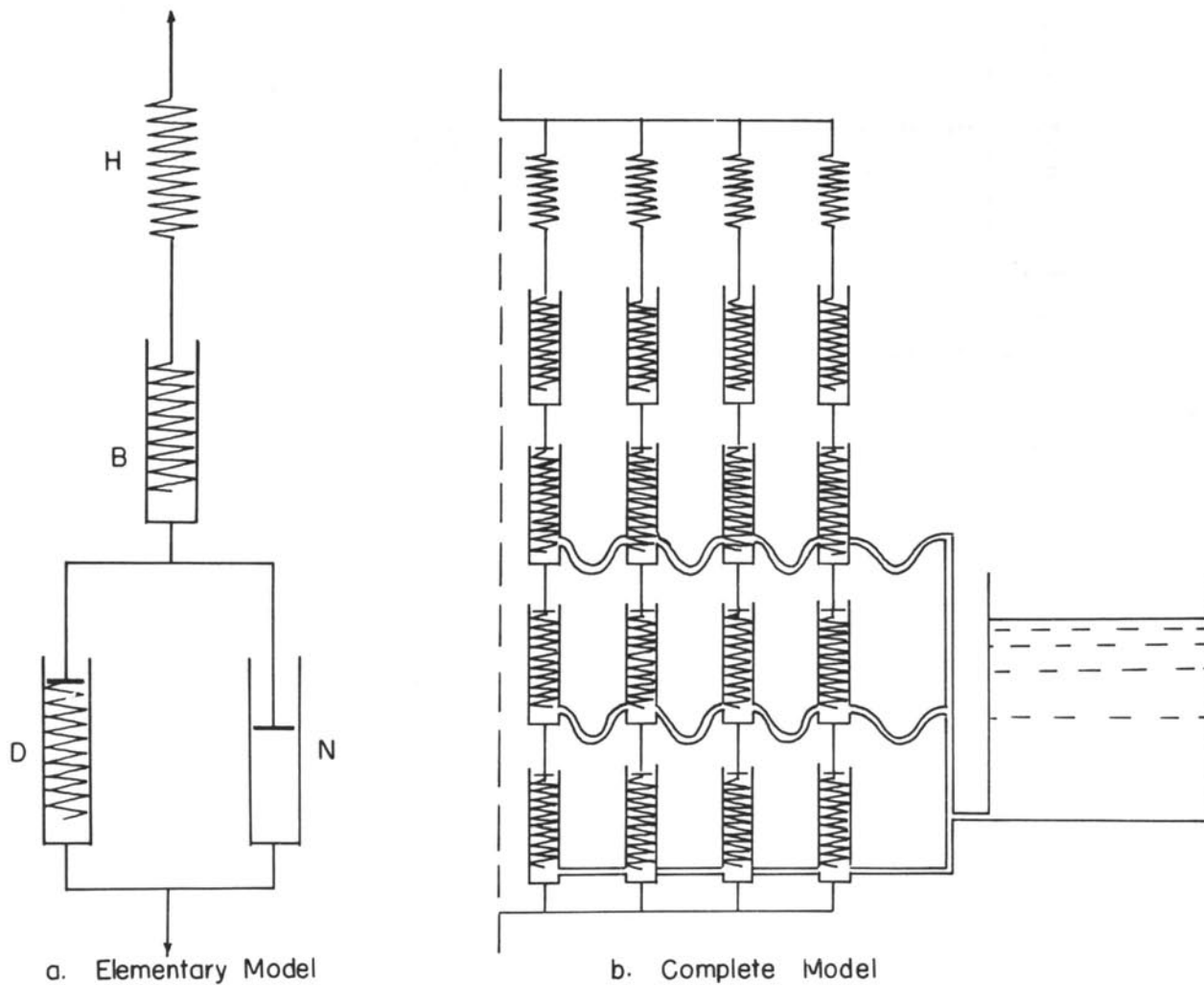


FIGURE 23. RHEOLOGICAL MODEL FOR CONCRETE AFTER TORROJA AND PAEZ

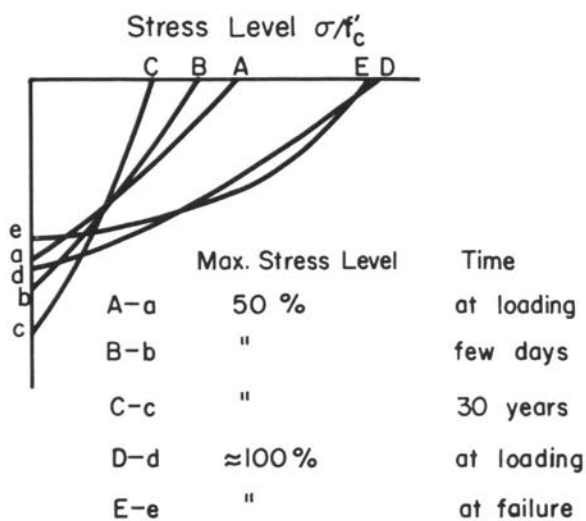


FIGURE 24. TIME-VARIATION OF COMPRESSIVE STRESSES ACROSS A RECTANGULAR SECTION UNDER FLEXURE

FIGURE 25. RELATIVE ISOCHRONIC  
STRESS-STRAIN CURVE FOR CONCRETE

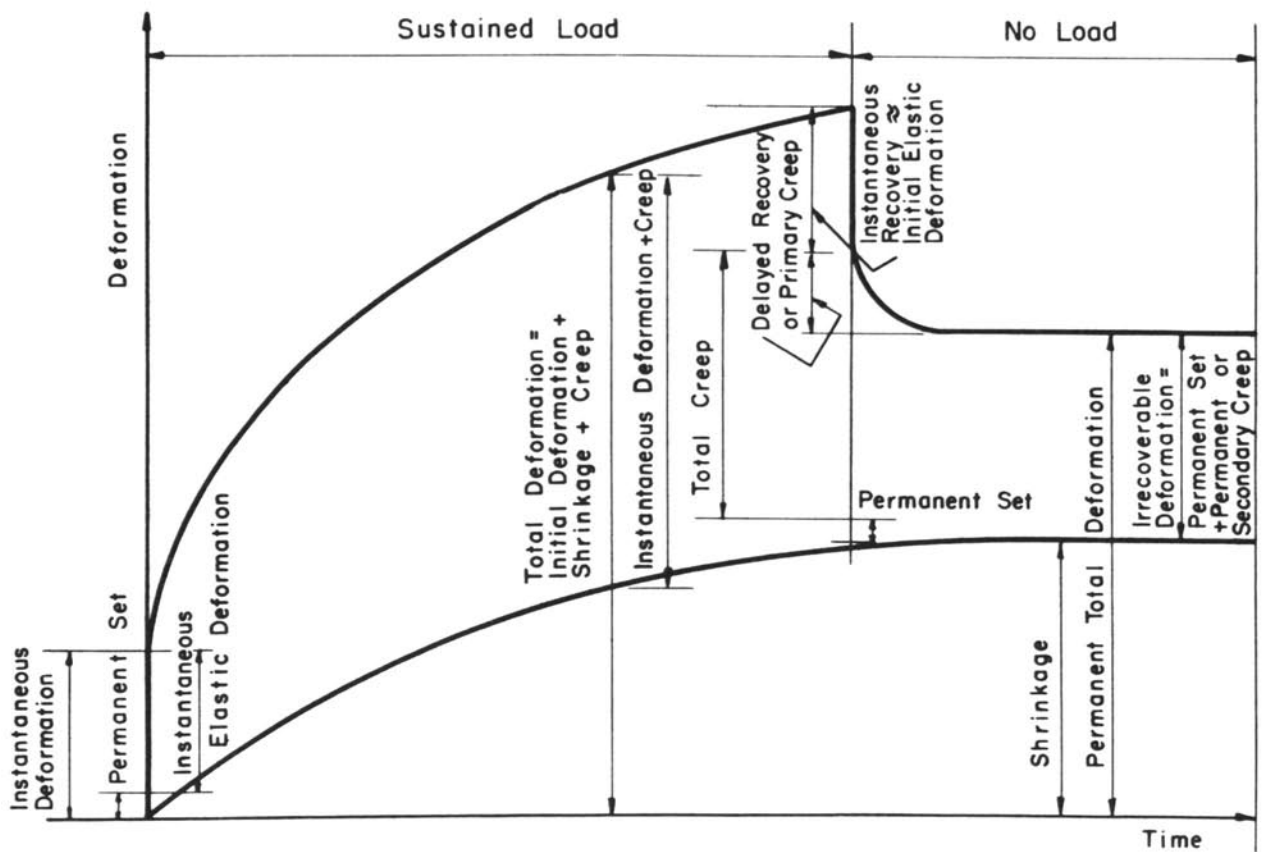
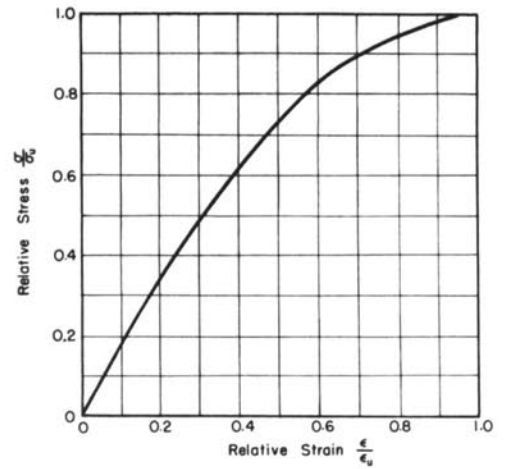


FIGURE 26. TYPICAL DEFORMATION V.S. TIME CURVE FOR CONCRETE UNDER CONSTANT LOAD

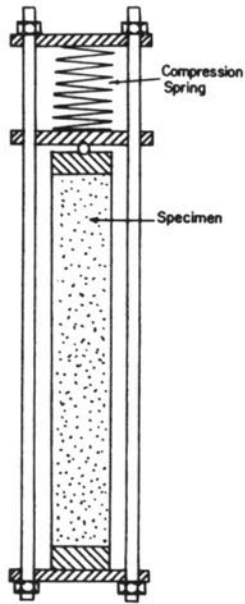


FIGURE 27. SPRING-LOADED  
COMPRESSION RACK

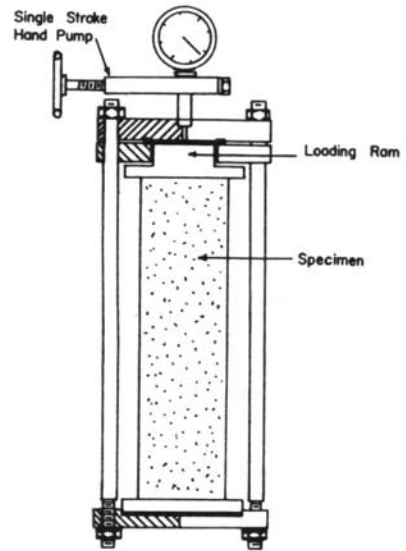


FIGURE 28. HYDRAULIC COMPRESSION RACK

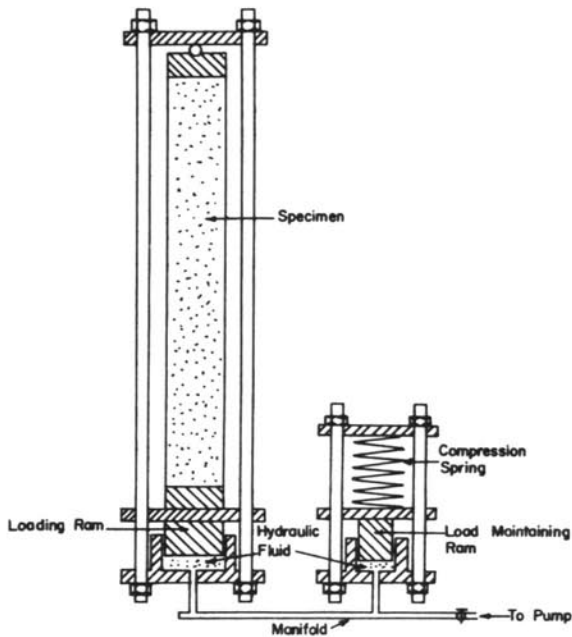


FIGURE 29. STABILIZED HYDRAULIC COMPRESSION RACK

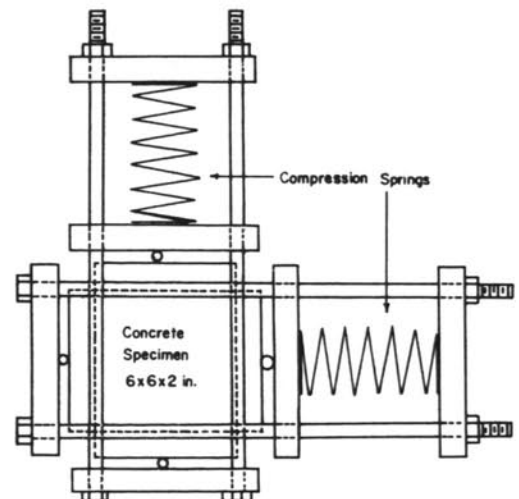


FIGURE 30. ARRANGEMENT FOR SUSTAINING  
BIAXIAL COMPRESSION

FIGURE 31. SUSTAINED TENSION  
PRODUCED BY INTERNAL PRESSURE

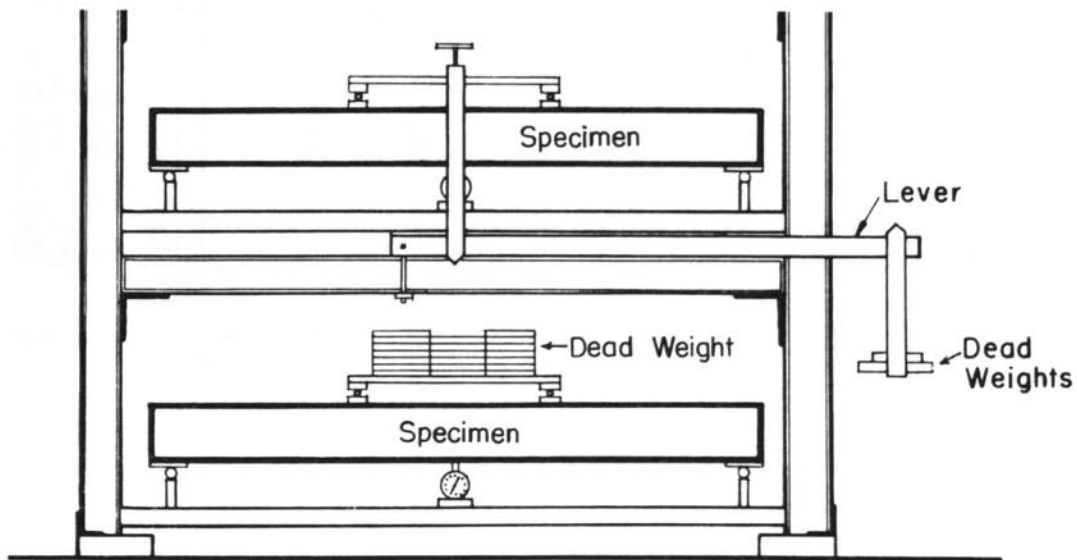
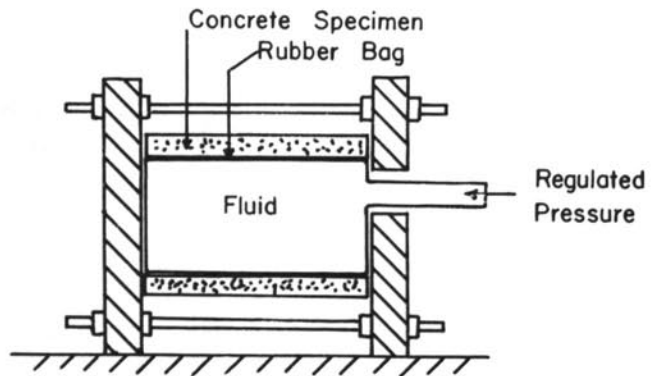


FIGURE 32. ARRANGEMENT FOR APPLICATION  
OF SUSTAINED FLEXURE

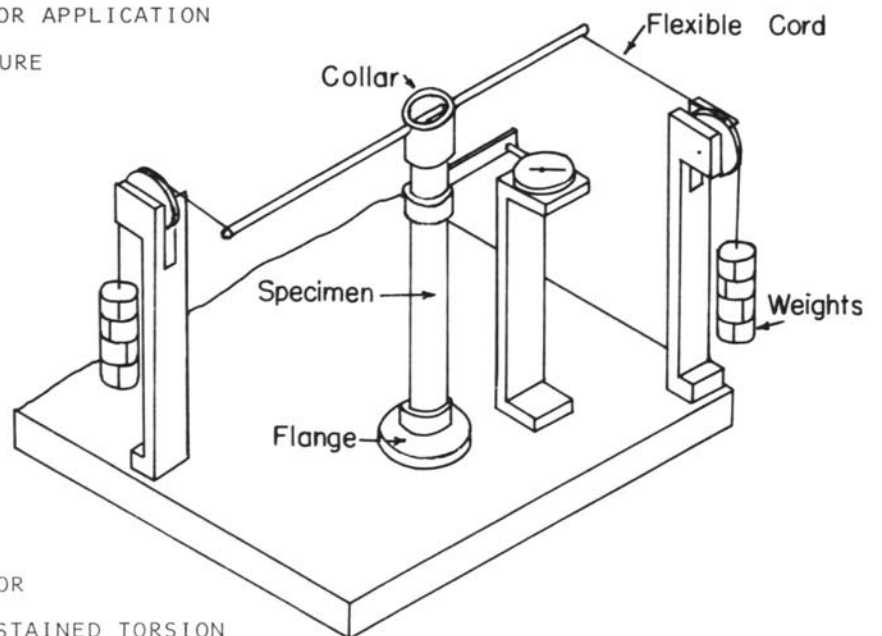


FIGURE 33. ARRANGEMENT FOR  
APPLICATION OF SUSTAINED TORSION

FIGURE 34. A SIMPLE MECHANICAL EXTENSOMETER

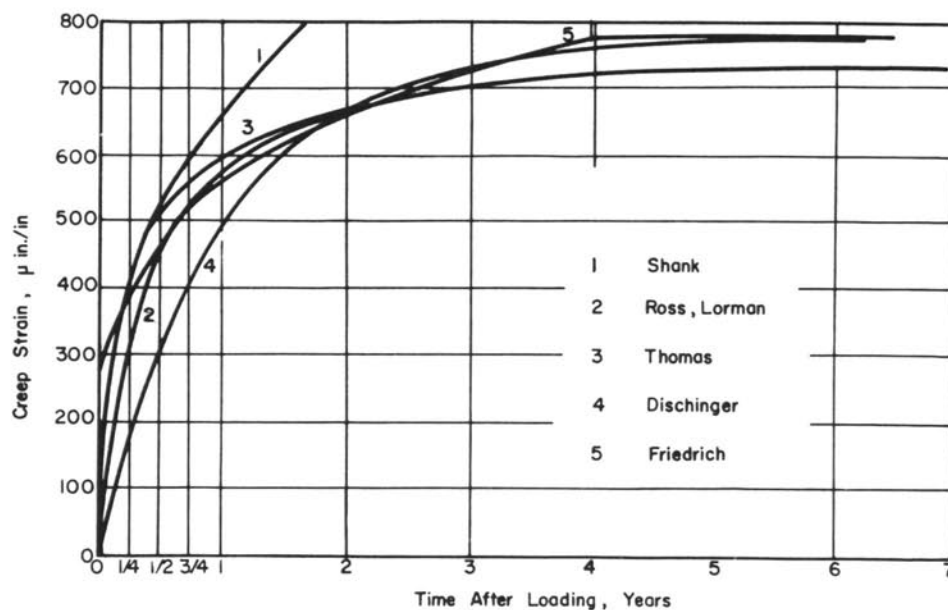
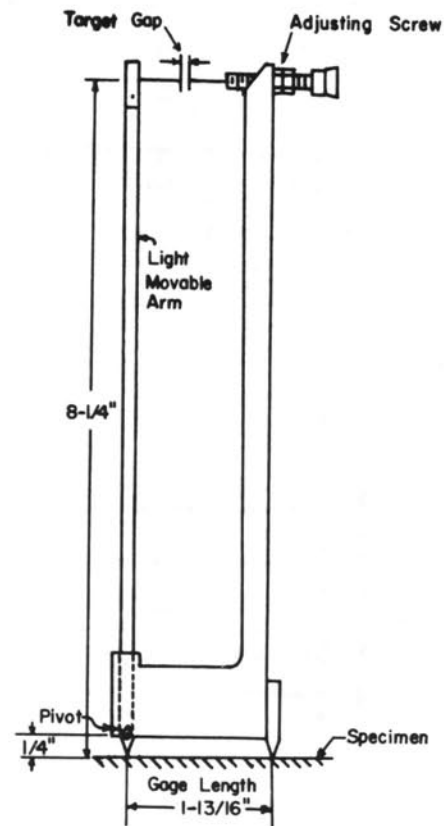
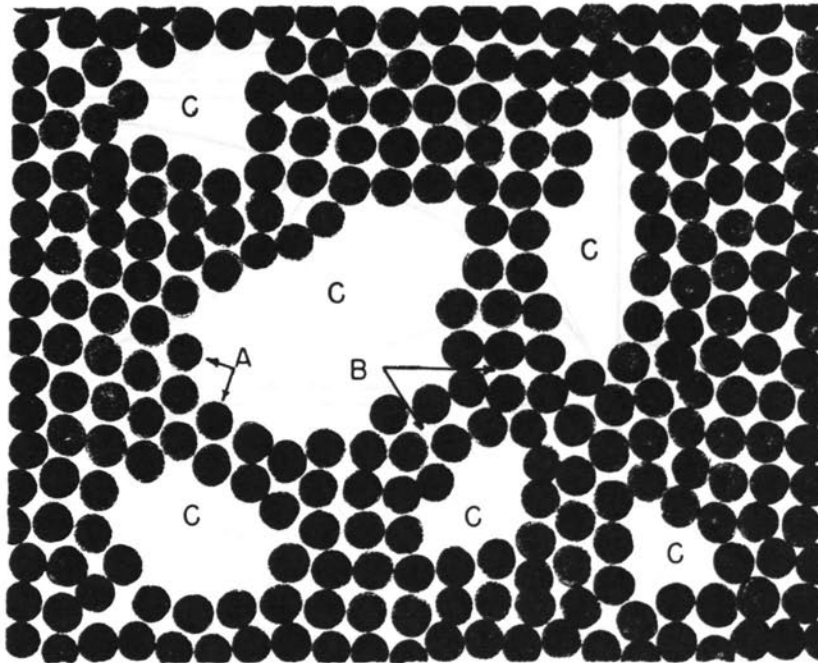


FIGURE 35. COMPARISON OF THE GENERAL FORM OF VARIOUS CREEP FUNCTIONS



A - Gel Particles

B - Gel Pores

← C - Capillary Pores

FIGURE 36. POWERS' SIMPLIFIED MODEL FOR CONCRETE PASTE

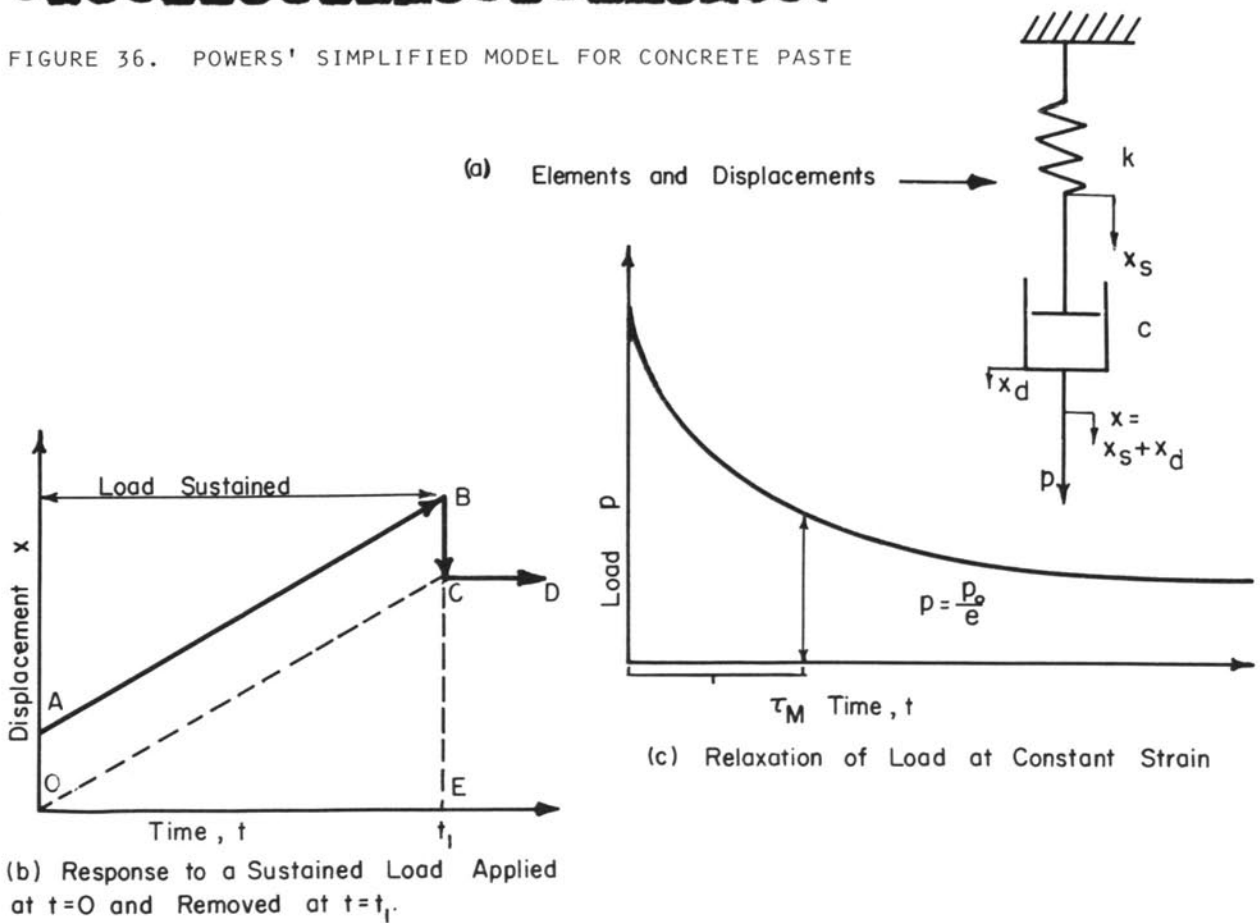


FIGURE 37. THE MAXWELL BODY AND ITS CHARACTERISTICS



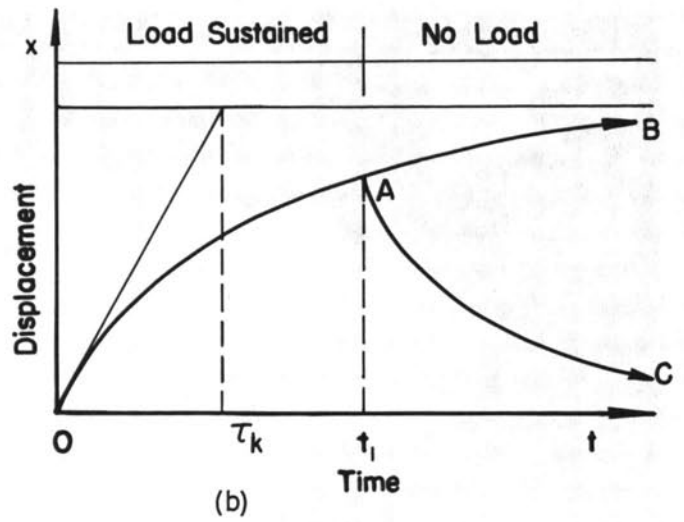
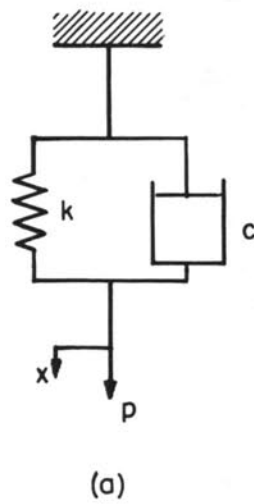


FIGURE 38. THE KELVIN BODY AND ITS RESPONSE

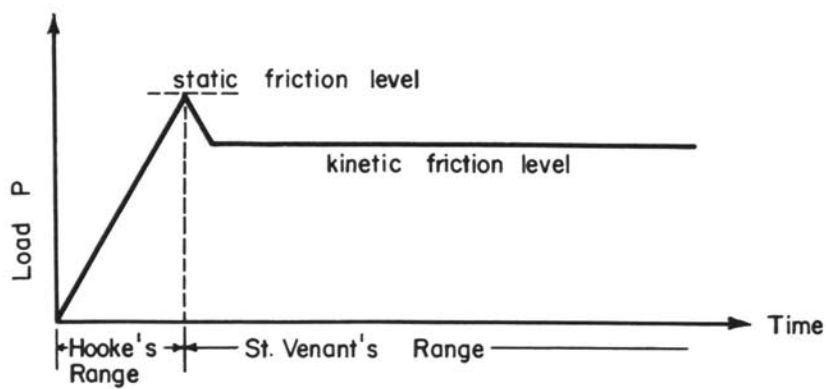
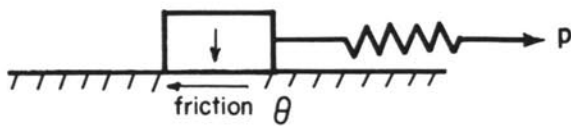


FIGURE 39. THE HOOKE-ST. VENANT BODY AND ITS RESPONSE

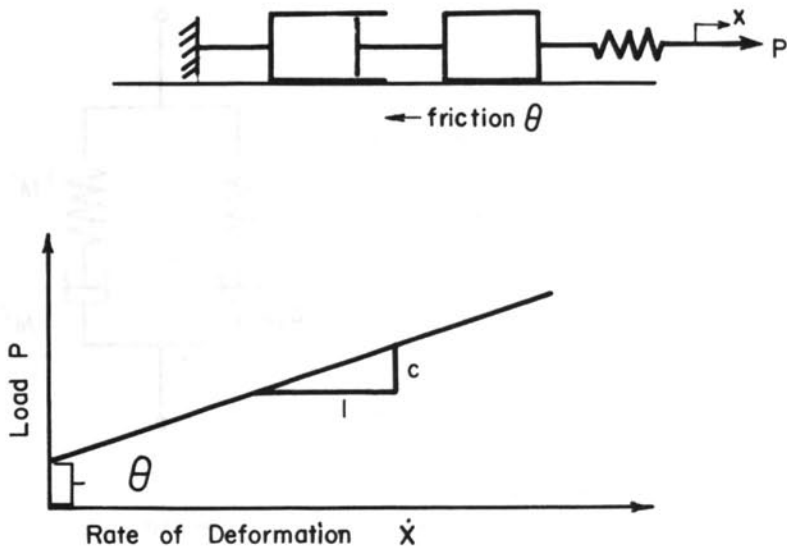


FIGURE 40. LOAD V.S. RATE OF DEFORMATION RELATION FOR THE BINGHAM BODY

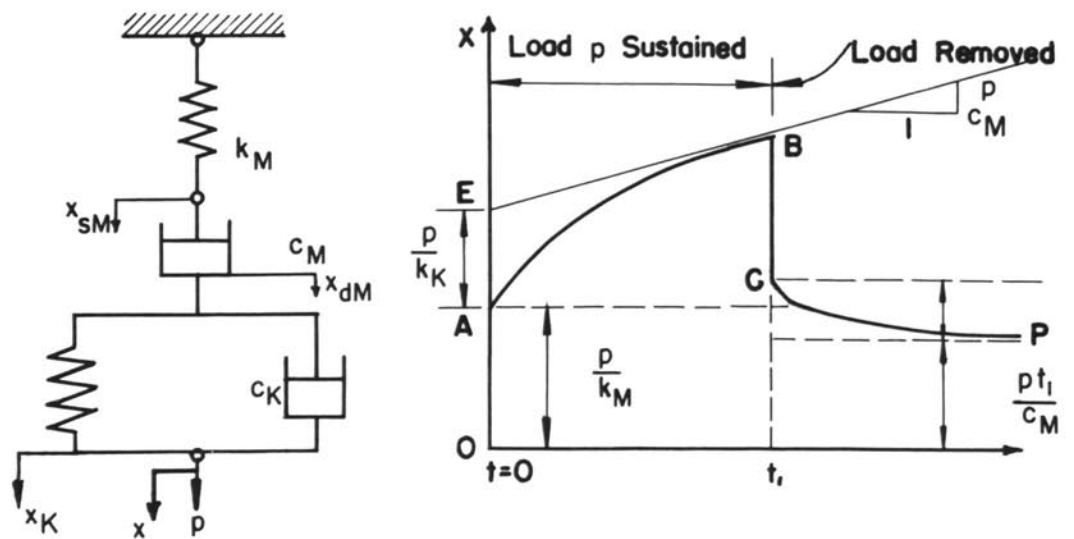


FIGURE 41. THE BURGERS BODY AND ITS RESPONSE

FIGURE 42. THE DOUBLE MAXWELL BODY

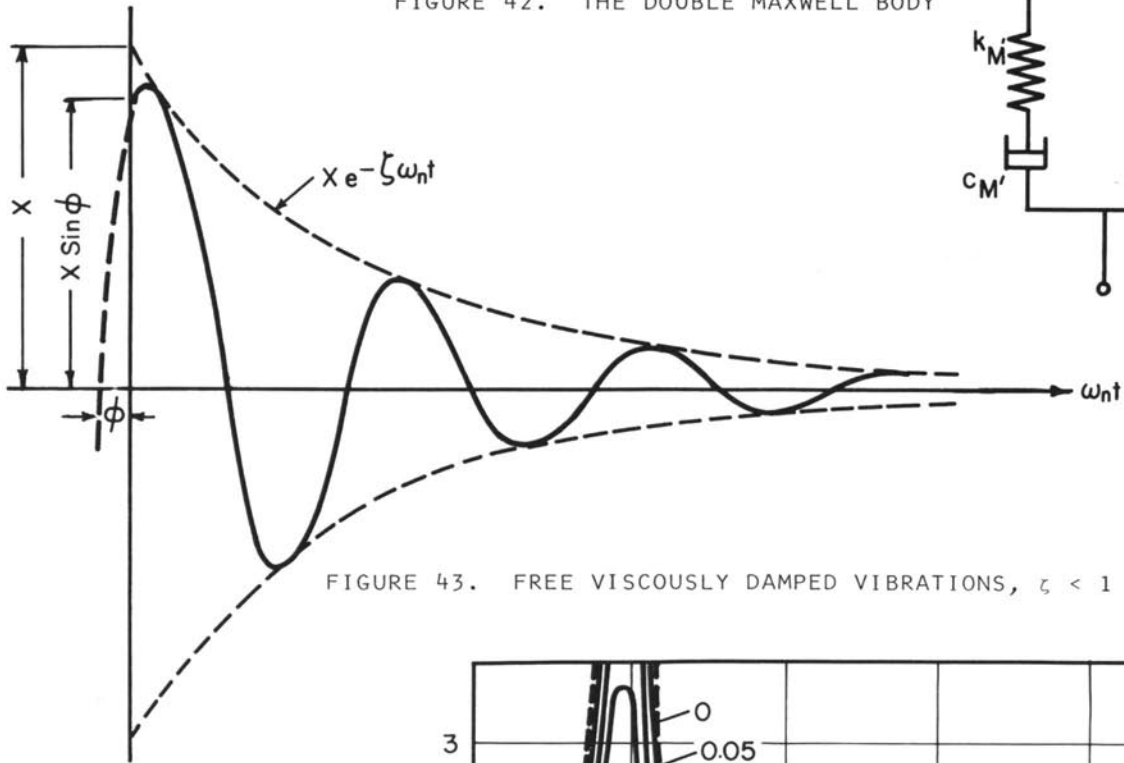
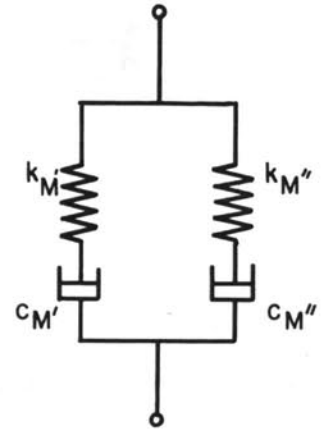


FIGURE 43. FREE VISCOUSLY DAMPED VIBRATIONS,  $\zeta < 1$

FIGURE 44. RESPONSE OF VISCOUSLY DAMPED SYSTEM UNDER FORCED VIBRATION

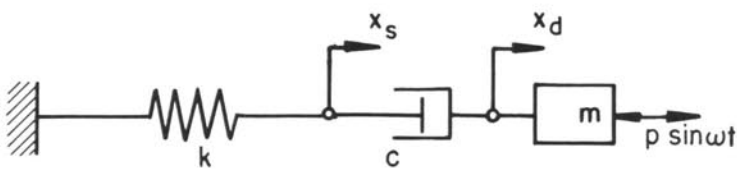
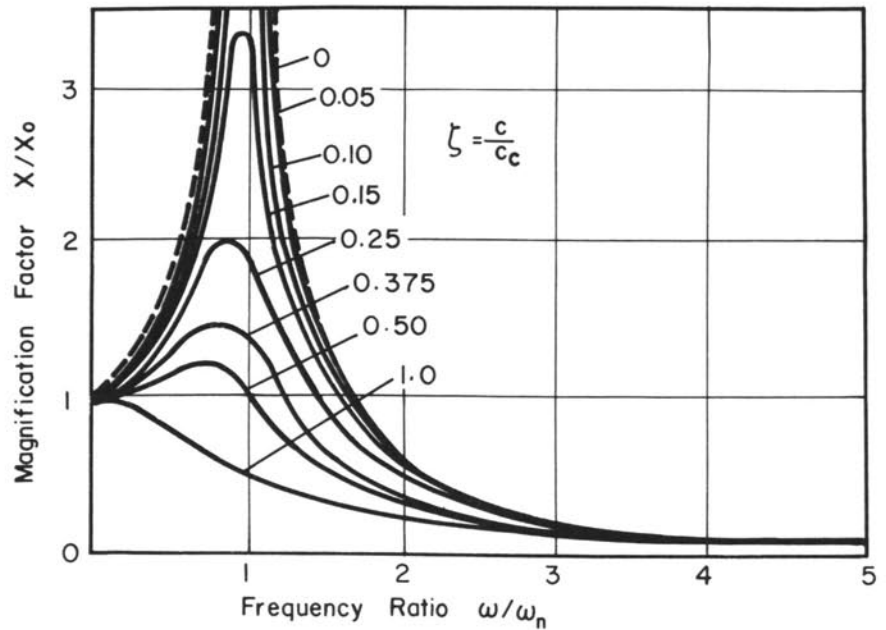
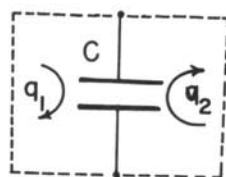
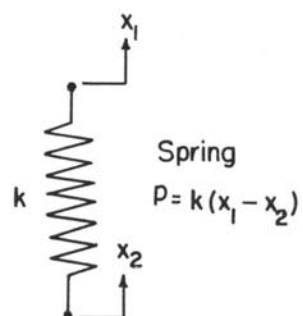
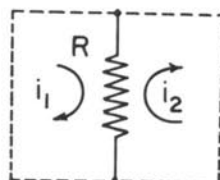
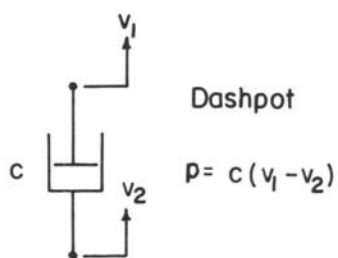


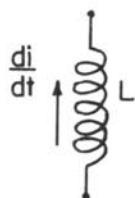
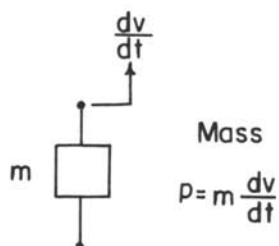
FIGURE 45. MAXWELL BODY UNDER A SINUSOIDAL LOAD



Capacitance  
 $e = \frac{1}{C}(q_1 - q_2)$

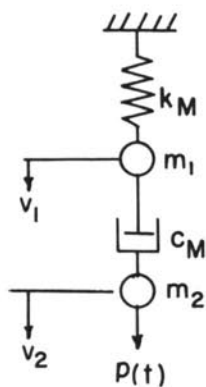


Resistance  
 $e = R(i_1 - i_2)$

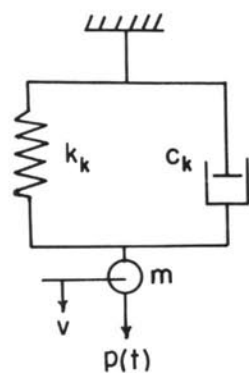
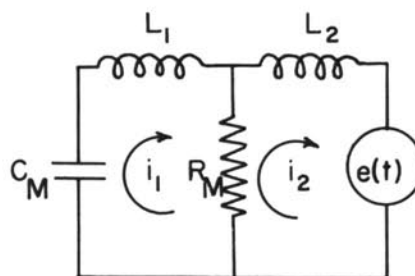


Inductance  
 $e = L \frac{di}{dt}$

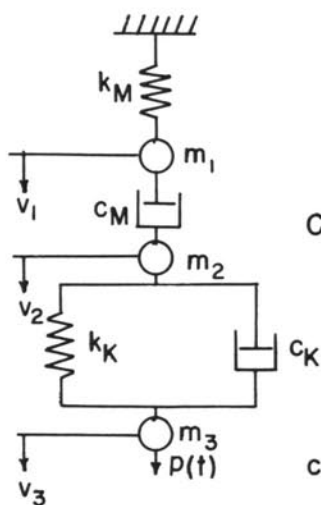
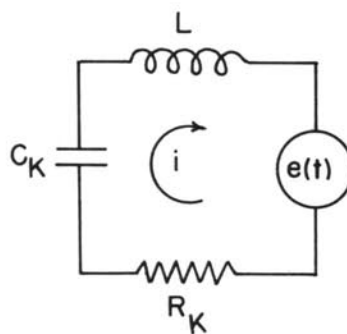
FIGURE 46. ELECTRICAL ANALOGS OF RHEOLOGICAL MODEL ELEMENTS



a. Maxwell Body



b. Kelvin Body



c. Burgers' Body

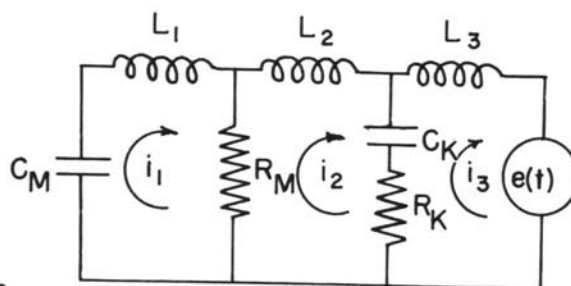


FIGURE 47. ANALOGOUS ELECTRICAL CIRCUITS FOR THE DYNAMIC RESPONSE OF RHEOLOGICAL MODELS

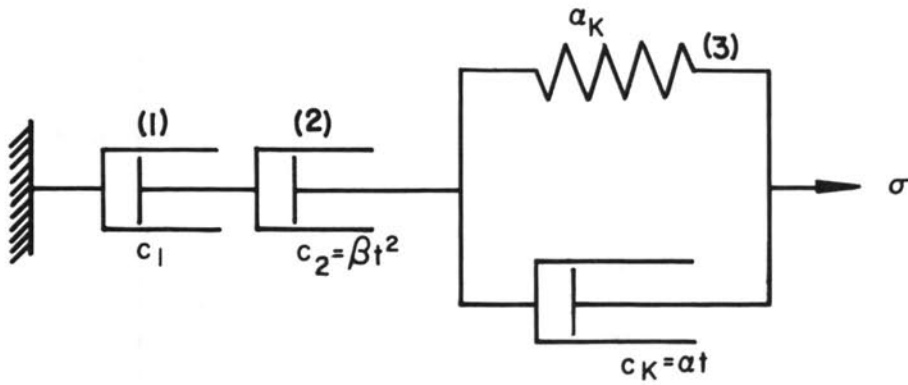


FIGURE 48. RHEOLOGICAL MODEL FOR CONCRETE AFTER FLÜGGE

FIGURE 49. RHEOLOGICAL MODEL FOR  
CONCRETE AFTER COWAN →

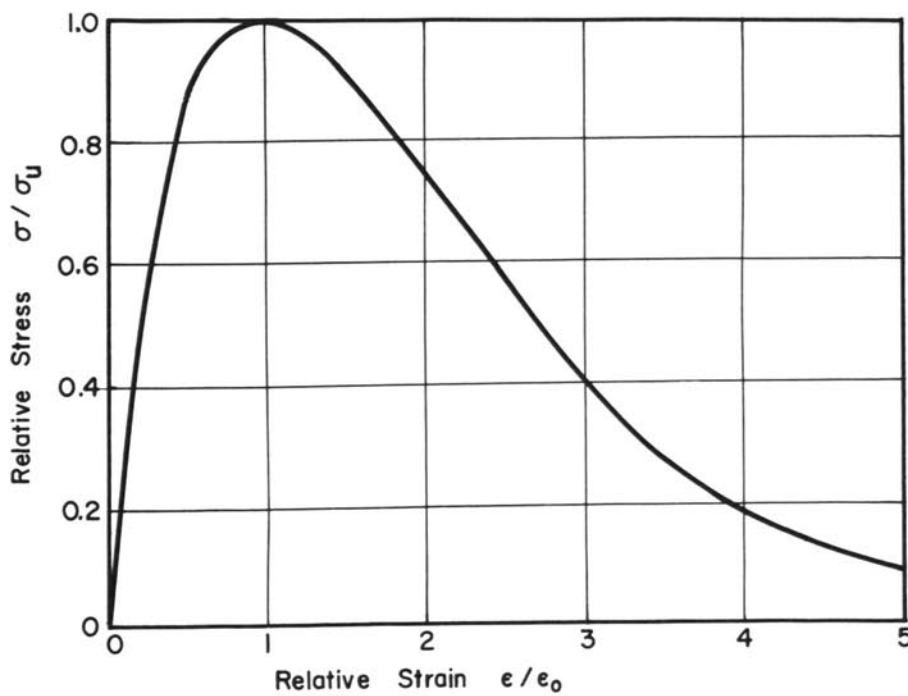
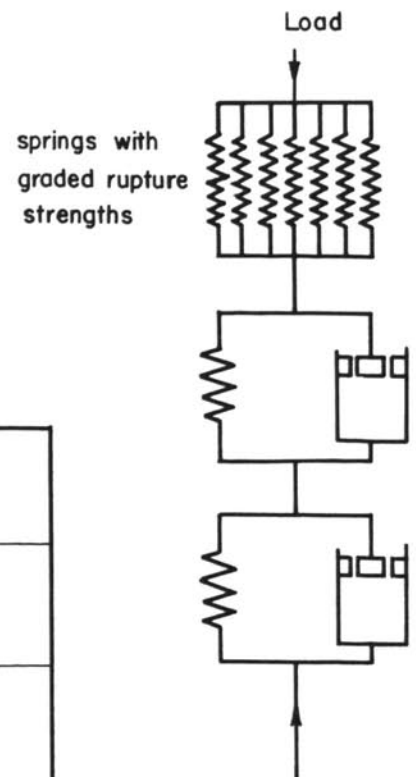


FIGURE 50. LOAD-DEFORMATION RESPONSE OF COWAN'S MODEL UNDER RAPID LOADING

FIGURE 51. RHEOLOGICAL MODEL FOR  
CONCRETE AFTER HANSEN

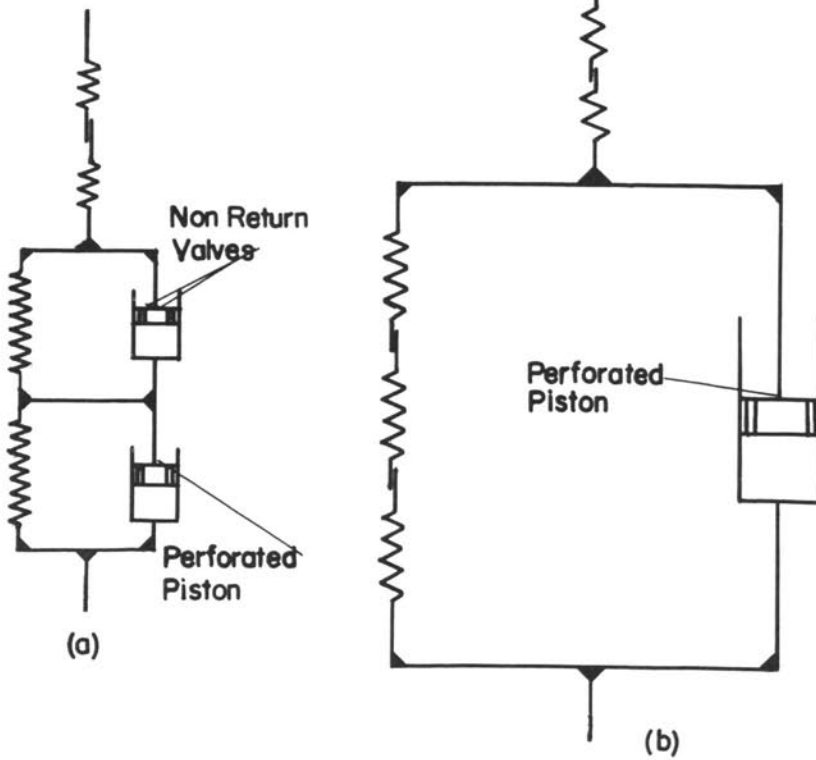
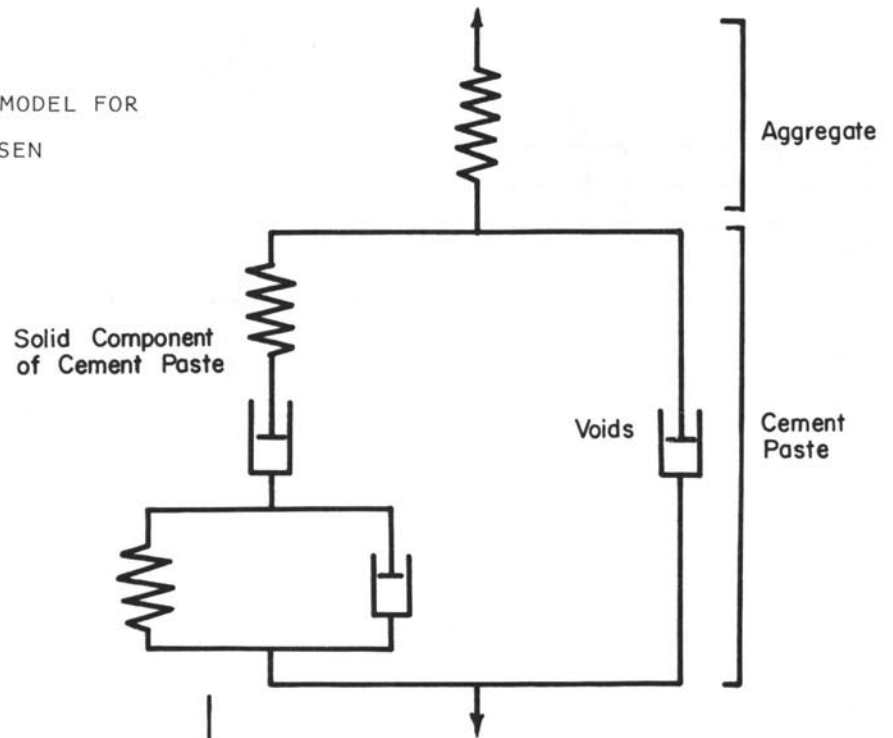


FIGURE 52. RHEOLOGICAL MODEL FOR HARDENED CEMENT PASTE AFTER GLUCKLICH



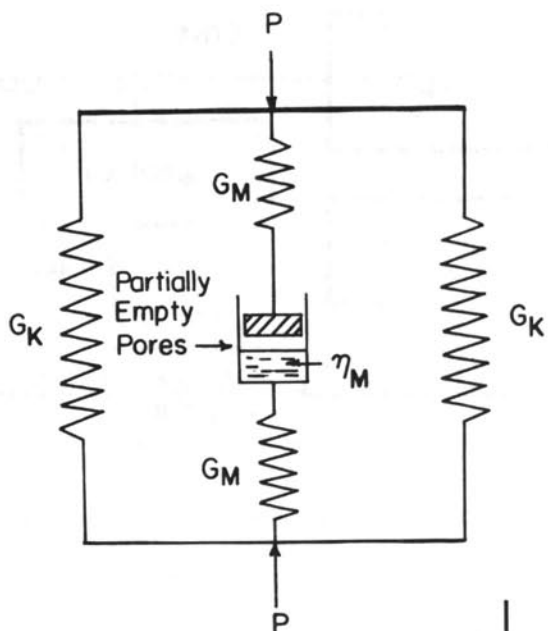


FIGURE 53. RHEOLOGICAL MODEL  
FOR MORTAR UNDER TORSION AFTER  
GLUCKLICH AND ISHAI

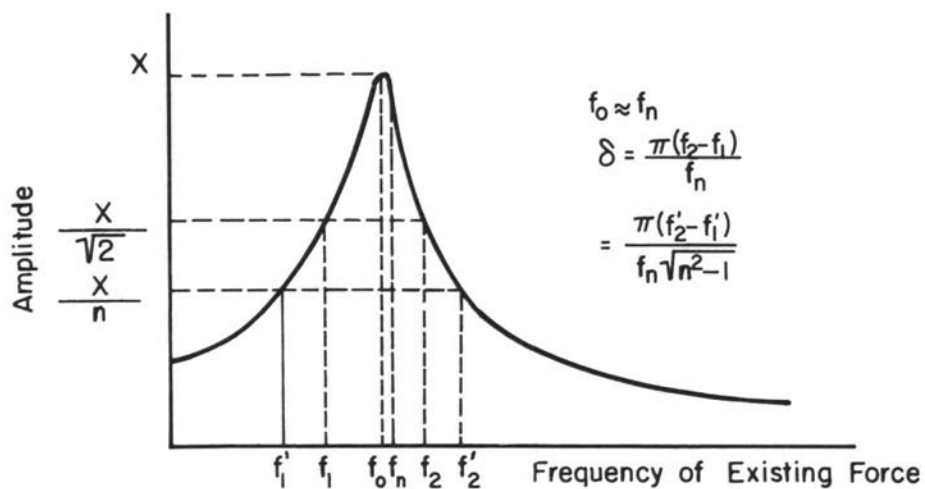


FIGURE 54. STEADY-STATE AMPLITUDE-FREQUENCY  
CURVE UNDER FORCED VIBRATION

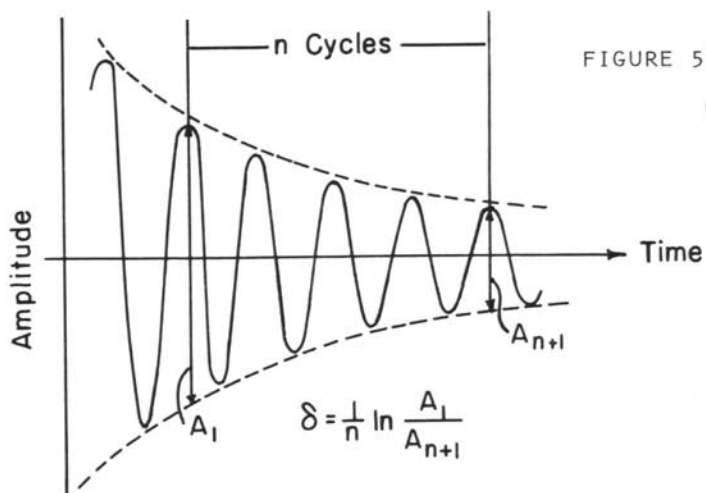


FIGURE 55. DECAYING AMPLITUDE CURVE

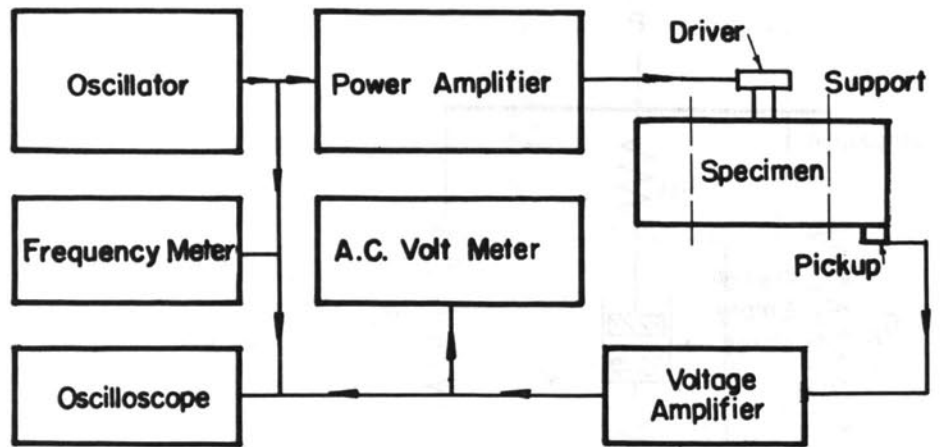


FIGURE 56. TYPICAL SETUP FOR CONSTANT AMPLITUDE SONIC TEST

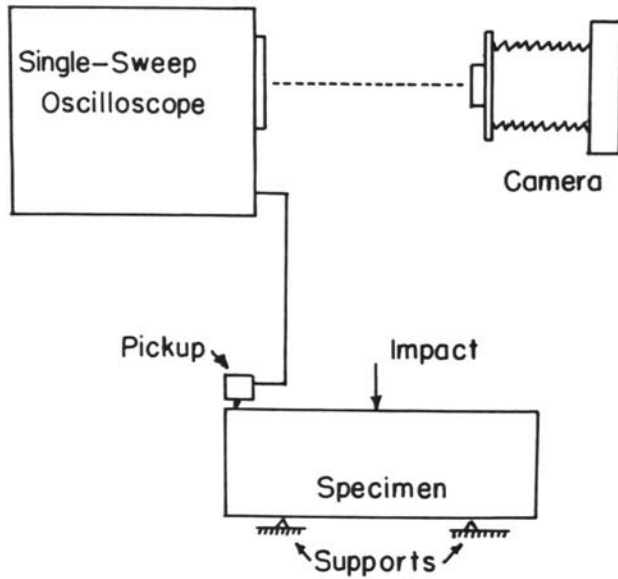


FIGURE 57. ARRANGEMENT FOR THE DECAYING AMPLITUDE TEST

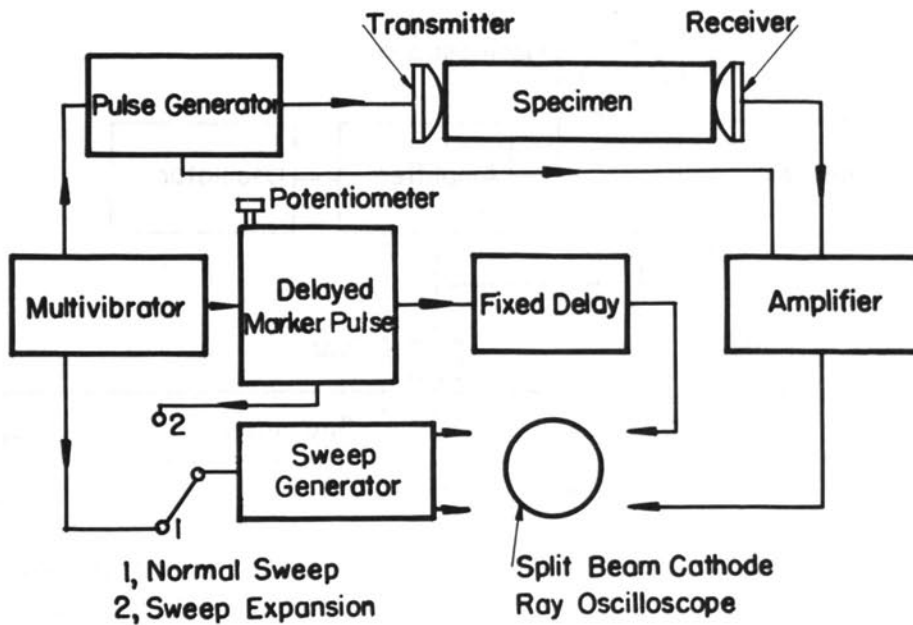


FIGURE 58. THE SONISCOPE FOR MEASURING PULSE VELOCITY

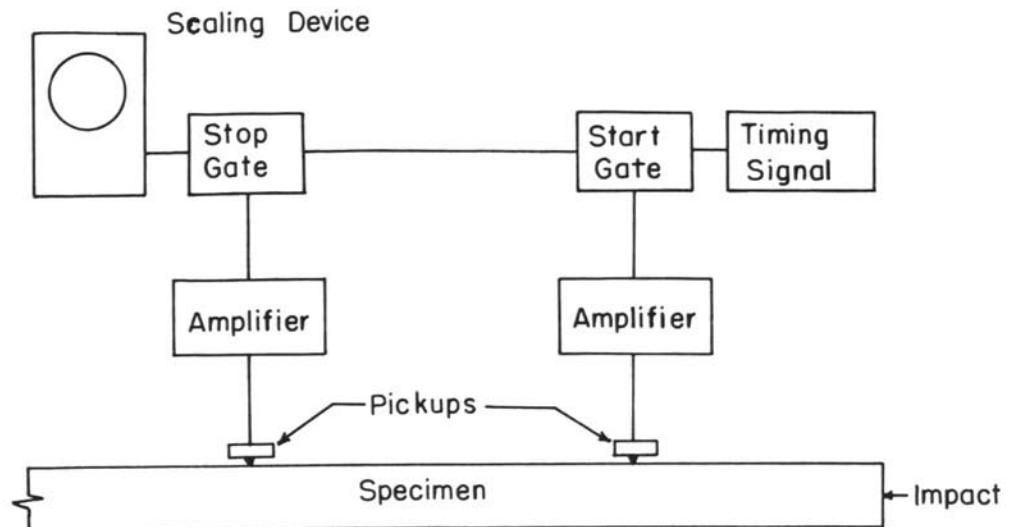


FIGURE 59. GATED INTERVAL-TIMER FOR MEASURING PULSE VELOCITY

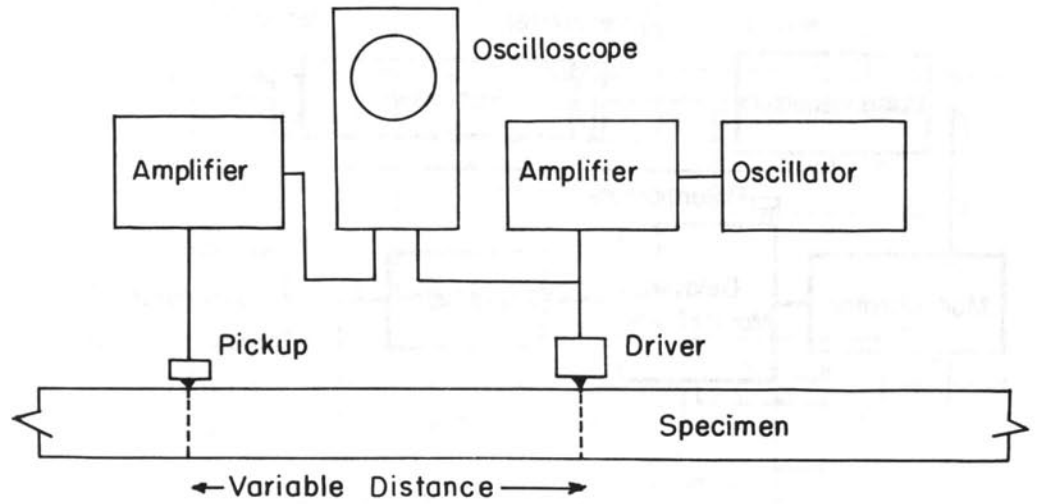


FIGURE 60. PHASE-SHIFT TECHNIQUE FOR WAVE VELOCITY MEASUREMENT

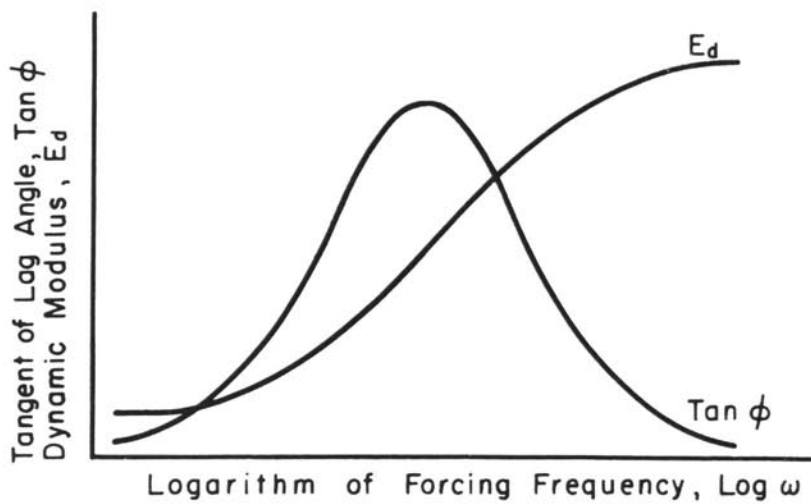


FIGURE 61. FREQUENCY-DEPENDENCE OF DYNAMIC MODULUS AND TANGENT OF LAG ANGLE FOR A THREE-PARAMETER MODEL

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## VI. ORIGIN OF CREEP

### A. THE NATURE OF CREEP

A question that often puzzles researchers in concrete is, "Why does concrete creep?" A large number of investigators have contributed to the solution of the problem of concrete creep, and several\* of them have also speculated as to the nature of creep. Nevertheless, the "why" of the phenomenon of creep has not yet been fully explained. While there are a number of points on which different investigators agree, there are others on which radically different views have been held, including a view categorically denying the existence of creep as an independent phenomenon.<sup>(37)</sup> This is a result of the complex nature of the phenomenon of creep and its association with the phenomena of shrinkage, elastic deformation, aging, moisture exchange with the environment, etc. Since the nature of hardened cement paste and the simpler phenomenon of shrinkage have not yet been completely deciphered, it is not surprising although it is unfortunate that the nature of creep has not been completely understood. However, to be able to unify available information on creep behavior of concrete and to achieve economy of effort in acquiring further and more reliable knowledge about the phenomenon so important from

a practical point of view, it is desirable to obtain some explanation of the phenomenon of creep which explains all, or at least most of the observed facts about creep of concrete and is consistent with the established knowledge of the day. Since certainty is not achievable, that hypothesis which explains most facts best has to be accepted as the correct one, at least for the time being.

The existence of creep in concrete is an empirical fact. Specimens of concrete under sustained stresses have shown time-dependent deformations in excess of what would have been expected from simultaneous observations of unloaded control specimens similar, as far as possible, to the loaded ones, except that they are not under external stress. This excess is defined as creep and studied as such. Any other way of defining creep would be equally valid. Terms like "true creep" and "actual creep" have often been employed by investigators who probably made the mistake of "begging the question;" these terms have to be defined first. Vexing again is the question of the interdependence of shrinkage and creep. Considering the preceding definition of creep, however, the question of interdependence becomes redundant (at least in the initial part of the inquiry into the nature of creep), but assumes importance when the interactions between alleged causes are analyzed. These issues cloud the problem

\* See L'Hermite,<sup>(74)</sup> Wagner,<sup>(77)</sup> Torroja,<sup>(66)</sup> Hansen,<sup>(78, 79, 80)</sup> Lea,<sup>(86)</sup> Lynam,<sup>(26)</sup> Lorman,<sup>(36)</sup> Maney,<sup>(37)</sup> Freudenthal,<sup>(56)</sup> Freysinet,<sup>(57)</sup> Neville.<sup>(69)</sup>

of creep and the meaning of the term remains vague. This, however, need not seriously hinder a qualitative analysis.

#### B. THE MECHANISMS OF CREEP

During a survey of the literature on creep, one comes across a variety of hypotheses which attempt to explain, wholly or partially, the mechanism of creep. These may be grouped under one or the other of the following categories:

- a. Crystalline flow
- b. Seepage or gel water flow
- c. Viscous flow of cement paste
- d. Delayed elasticity
- e. Surface tension effects
- f. Tendency towards maximum stability
- g. Nonuniform shrinkage
- h. Internal rupture

As none of these explanations, or any others for that matter, can fully account for all aspects of the observed behavior, it may be said that no "theory" of creep exists; in a strict scientific sense, hypotheses do. These hypotheses may, of course, be referred to as theories in the broader sense of the term. In the following, these theories will be reviewed critically and evaluated in the light of available supporting evidence.

##### 1. Crystalline Flow

Plasticity of metals has been attributed to the phenomenon of slip in crystals.\* This might suggest that the phenomenon of creep in concrete could be of a similar origin. The term "plastic flow," which was very frequently used for "creep" in earlier papers on creep, probably had its origin in the similarity of creep to plasticity of metals.

When a crystal of metal is subjected to stress, it undergoes a plastic deformation, provided the stress is above the yield point

of the crystal. The plastic deformation is a result of slip along a critical plane in the crystal lattice. During this plastic deformation, no volume change takes place.

Lynam<sup>(26)</sup> rules out this phenomenon as a possible cause for creep in concrete because concrete is much stronger in shear than in tension; hence concrete would be expected to fail by separation rather than by shear slip.

In addition, plastic flow would not be expected to occur below a certain threshold value of stress. As observed by Glanville<sup>(21)</sup> there is probably no observable threshold value below which creep in concrete is non-existent; this fact apparently makes plastic flow improbable as a cause of creep in concrete. On the other hand, it is possible that there may be a continuous distribution or spectrum of yield values starting from very low values, in which case no sharp yield point would exist.

According to Glanville,<sup>(21)</sup> creep at lower stress levels might be essentially viscous in nature, but at higher stresses crystalline slip could be a significant factor. However, Neville<sup>(69)</sup> refers to the observations by Shank,<sup>(106)</sup> and Jensen and Richart,<sup>(34)</sup> who found creep curves for low and high stresses to be essentially similar in general nature.

At higher stresses, creep behavior of lean concrete (in which the aggregate predominates) resembles plasticity, but this is different from plastic flow of metals. In concrete the behavior is perhaps due to the rupture of bonds between aggregate particles and the cement paste, which are not re-established. In crystalline slip in metals, the broken bonds continually re-establish themselves by virtue of strong cohesion.<sup>(56)</sup>

For stress levels from 25 to 50 per cent of the ultimate strength of concrete, stress and strain after a certain time have been

\*See Chapter 5 by Burgers in Houwink's reference.<sup>(140)</sup>

observed to be almost directly proportional. This phenomenon would be improbable if creep strains were due to plasticity of crystals. There is, on the other hand, a marked departure from proportionality at higher stress levels which suggests that at these levels, deformations of the plastic type may occur.

Thus, the hypothesis describing creep as a plastic deformation appears to be worth considering for higher stresses only, if at all.

The somewhat different phenomenon of intercrystalline slip has been suggested by Hansen,<sup>(79)</sup> as a possible source of viscous deformation. Disordered lattices developed at the junctions of adjoining metallic crystals are known to exhibit viscous flow and consequent stress relaxation.<sup>(141)</sup> A similar deformation at the contacts between cement gel particles and unhydrated grains of cement is envisaged.

## 2. Seepage of Gel Water Flow

Lynam<sup>(26)</sup> was probably the first to propose seepage of gel water as the cause of creep in concrete. It has since been suggested by other authors, notably Lea,<sup>(86)</sup> Seed,<sup>(48)</sup> Lorman,<sup>(36)</sup> and Hansen.<sup>(78)</sup>

If hardened cement paste is considered to be composed of a colloidal gel, creep in concrete can be shown to be consistent with the phenomenon of seepage of gel water under pressure, a phenomenon observed in other colloidal gels. Under pressure, the adsorbed water flows into the capillary spaces, from whence it may flow out and evaporate. Accordingly, creep is analogous to drying shrinkage except that the gel water movement is caused by pressure instead of differential hygrometric conditions between the specimen and the environment. This concept is supported by the striking similarity between creep and shrinkage curves and their dependence on practically

the same physical factors, i.e., water-cement ratio, type and fineness of cement, mix proportions, compaction, curing and storage conditions, and degree of hydration. Lynam considers it impossible and pointless to try to separate shrinkage and creep effects.

Lea and Lee<sup>(86)</sup> describe the theory as follows. Application of stress leads to an alteration in the gel vapor pressure and hence the gel water content, with accompanying volume changes. Hardened cement paste being essentially a rigid gel, equilibrium exists between the swelling pressure of the gel and the solid skeleton. The change in gel water content is due to the disturbance of the equilibrium. Creep, according to the seepage theory, is caused by the delay in re-establishing the equilibrium between the gel and its surroundings. According to Lynam, the rate of seepage would depend upon the moisture gradient; the steeper the gradient, the quicker the flow.

According to this theory, creep recovery is due to the tendency to re-establish the original state of affairs. If, during the period of sustained load some water has been driven out of the specimen, the recovery will not be complete. Lack of complete recovery may also be due to the formation of new bonds or due to gel particles taking up more stable positions. The part played by chemically combined water and the capillary water in the creep phenomenon is considered insignificant since creep effects are attributed almost entirely to the gel water movement.

Several objections and counter objections have been advanced for and against the seepage theory. Maney<sup>(37)</sup> postulated that if shrinkage and creep were due to the same phenomenon of movement of gel water, specimens under sustained loads would show a considerably greater moisture loss than corresponding unloaded



specimens. Tests carefully performed by him showed no marked difference in the weight of loaded and unloaded specimens of concrete. On the basis of this he concluded that creep could not be due to loss of water from the specimen to the environment. One of the objections advanced was that the test might not have been sensitive enough to record the difference. The counter objection maintained that the moisture losses for shrinkage and creep should have been comparable in view of the comparable magnitude of observed creep and shrinkage strains.

L'Hermite<sup>(74, 107)</sup> points out that under load a hygrometric gradient is created between the gel pores and the capillary spaces. The moisture movement corresponding to creep is thus almost entirely internal to the specimen; hence the absence of any measurable loss of weight. Shrinkage, on the other hand, entails direct moisture exchange with the external environment.

According to the seepage theory, creep in tension would be expected to be greater than compressive creep for concrete under water. Some tests have, on the contrary, reported about the same order of magnitude for both.

### 3. Viscous Flow of Cement Paste

The theory describing creep as a viscous flow essentially postulates that creep movement observed in hardened cement paste, mortar, and concrete is essentially due to a viscous movement of the cement gel under stress. Accordingly, cement paste is considered to be a highly viscous liquid, the viscosity of which increases with time.

Freudenthal<sup>(56)</sup> states; "Concrete is formed by an aggregation of loose grains (sand and aggregate) held together by a highly viscous liquid, the cement paste. The viscosity of the liquid increases with time, as a result of chemical changes within the structure

(crystallization) until a complete crystalline network blocks all viscous deformation. The relative volume of grains and of the viscous medium determines the mechanical behavior of concrete."

Reiner<sup>(49)</sup> also maintains that creep is due to the cement paste only, while the aggregate particles merely act as inert bodies. Neville,<sup>(69)</sup> on the other hand, argues that aggregate cannot be considered as inert because were it so, creep of a cement-aggregate system would be inversely proportional to the aggregate-cement ratio, which is not borne out by observations.

The phenomenon of creep in concrete was first attributed to a viscous flow of the paste by Thomas<sup>(24)</sup> and Glanville.<sup>(35)</sup> Thomas attributed the progressive reduction in the rate of creep to the gradual transfer of load from the cement paste to the inert aggregate. According to Glanville, the creep taking place at low loads is essentially due to viscous flow.

An argument based on observed volume change is often advanced against the viscous flow theory of creep. If creep were due to viscous flow, it is argued, the volume of the specimen would remain constant. But on the contrary, observed lateral deformations in specimens loaded longitudinally have been found to be only a fraction of what they would be in case of a constant volume phenomenon. Ross<sup>(67)</sup> found no time-dependent lateral strains accompanying longitudinal strains under sustained stresses. This decrease of volume is considered by Reiner<sup>(142)</sup> to be an example of "volume" or "isotropic" flow. He describes creep as a viscous flow, in which the apparent decrease of volume is due to the material flowing into its own pores.

Viscous flow requires both a proportionality between stress and isochronic strain,

and between stress and strain rate, and a finite rate of strain for a finite stress of however small a magnitude. Creep observations on concrete substantiate both of these requirements to a certain degree. For stress levels up to about 25 to 50 per cent of the ultimate strength, creep has been observed to be nearly proportional to the sustained stress and takes place under sustained stresses as low as one per cent.

#### 4. Delayed Elasticity

Creep in concrete is often considered to be a delayed elastic phenomenon resulting from one or more causes. The phenomenon is well-known in the case of other materials and goes under the names of elastic aftereffect<sup>(49, 140)</sup> or anelasticity.<sup>(143)</sup> If a load is suddenly imposed on a firmo-viscous body<sup>(49)</sup> (i.e., a body with a solid elastic skeleton with its voids filled with a viscous fluid), the viscous fluid will tend to delay the elastic response of the solid skeleton by taking up load and gradually transferring it to the solid skeleton. The phenomenon is typical of a Kelvin body under a sustained load. A similar effect in the case of concrete and cement paste has been noted by several investigators. In the case of concrete, the elastic deformation of a solid skeleton formed by the aggregates and the gel crystals may be delayed by the viscous phase made up of the gel or capillary water. Another possibility would be to consider that the aggregate forms the elastic skeleton and the cement paste acts as the viscous phase.

Hansen<sup>(79)</sup> suggests that the delayed elasticity could be a manifestation of a diffusion process. The external load induces molecular orientation in the gel particles, which tends to decrease the configurational entropy. Gradual restoration of this decreased entropy through molecular diffusion results in

time-dependent elastic movements. Segmental diffusion among the crumpled foil-like components of the gel may also occur. Increase in the ambient temperature would tend to accelerate the diffusion process. Increase in the creep rate with temperature is well known.

Depending upon whether the viscous phase is able to resume its original spatial distribution, the phenomenon may or may not be reversible. In the case of concrete the phenomenon of creep is partly reversible; this could be due to a partial return of the viscous phase to its original distribution.

Another type of elastic aftereffect is involved in the explanation of creep on the basis of Freyssinet's capillary water theory. (66, 57) According to this hypothesis, in a specimen under sustained compressive stress in one direction, the pores are constricted in the direction of the force and are dilated in the normal directions; the pore volume as a whole decreases as Poisson's ratio is smaller than one-half. The average meniscular radius will increase, thus causing a reduction in the so-called surface tension forces; this reduction will cause the specimen to dilate. The effect will be opposite to that of the load, hence the specimen will, in effect, show less shortening under load than what it would have shown in absence of the effect. However, the water in the pores will tend to escape gradually from the pores until an equilibrium with the surroundings is established. This causes gradual deformation in the direction of the sustained stress, which is creep. Thus, according to this theory, the creep effect is a delayed elastic effect caused by the changes in average meniscular radii of the pores in the cement gel. On removal of load, the phenomenon may be expected to be reversed, thus exhibiting recovery, which may or may not be complete

depending upon whether all the water is accessible again.

A somewhat different elastic aftereffect was conceived by Maney.<sup>(37)</sup> According to him, all the time-dependent deformation results from shrinkage. The excess of time-dependent deformation observed in the case of specimens under sustained load compared to that of specimens with no sustained load, is due to delayed elastic deformation resulting from nonuniform shrinkage. Although his explanation of creep involves several arbitrary assumptions regarding variation of shrinkage, magnitude of load, etc. and has been questioned,<sup>(113)</sup> such a phenomenon cannot be categorically denied without further verification. Until otherwise concluded, a part of what is measured as creep may very well be a result of the "Maney Effect" if it may be so termed.

#### 5. Surface Tension Effect

A delayed elasticity theory of creep based on Freyssinet's concept was briefly described above. Since the effect is caused by surface tension not involved in other theories, it warrants a separate classification. Explanations of creep based on the surface tension concept have also been discussed by Lea,<sup>(86)</sup> Torroja,<sup>(66)</sup> and Wagner.<sup>(77)</sup> The latter also describes Gehler's aging hypothesis supplementing the theory of Freyssinet. Torroja<sup>(66)</sup> points out that, according to Freyssinet's theory, the specimen should, on desiccation, start swelling again. However, he concedes that this extrapolation has little significance. Considering physical models of finite dimensions, Wagner<sup>(77)</sup> questions the applicability of the relations to the capillaries in cement paste, which are only a few millionths of a millimeter, in diameter.

#### 6. Tendency Towards Maximum Stability

While Freyssinet<sup>(57)</sup> explained the reversible part of creep as an effect of the delayed

elasticity caused by surface tension in water in capillaries of hardened paste, he considered the irreversible aspect of creep as due to the tendency of the whole system towards maximum stability under sustained load, which increases the probability of particle rearrangements. It is quite possible that a part of the irreversible deformation may be caused by this effect. Bletzacker<sup>(152)</sup> has suggested an approach for the study of the rheological properties of concrete based upon the laws of conservation of energy and the thermodynamic concept of entropy which defines the irreversibility of a mechanical process.

#### 7. Nonuniform Shrinkage

A case of nonuniform shrinkage was described earlier as the "Maney Effect." Another effect which, because of nonuniformity of shrinkage may affect long-time deformations of concrete under sustained loads, has been described by Pickett,<sup>(38)</sup> and may be called the "Pickett effect." This effect has been discussed in detail by Hansen.<sup>(78, 79)</sup>

It is well known that the stress-strain relationship in concrete departs significantly from proportionality for stresses greater than about half of the ultimate strength. Hence, if a concrete specimen is simultaneously subjected to two stress intensities, the resulting strains cannot be obtained by algebraic summation of strains corresponding to the individual stress intensities if either of the stresses or the stress corresponding to the algebraic summation of the individual stresses falls in the range beyond the proportional limit. Now, if a specimen of concrete is drying out, internal stresses due to nonuniform shrinkage will be set up until the specimen attains moisture equilibrium with the environment. If the load is sustained on a specimen so that after superposition of the load stresses on the internal stresses due to



nonuniform shrinkage, the resultant stress at any section becomes significantly greater than the proportional limit. The resultant strains obtained do not represent the summation of creep and shrinkage strains. Thus, for high stress levels and specimens with nonuniform moisture condition, a part of the measured creep may be due to the "Pickett effect." Although this has nothing to do with the "why" of creep, it does have relevance to creep as measured.

#### 8. Internal Rupture

It is conceivable that under sustained stresses a local breaking of bonds within or between particles in the hardened paste or between the paste and aggregate may take place in a concrete specimen. This may be responsible for a part of the instantaneous as well as the time-dependent non-recoverable deformations observed under sustained loads. According to Freudenthal,<sup>(56)</sup> deformation occurs and proceeds in lean concrete at a constant stress as it does in a plastic material; this apparent plasticity is, however, due to destruction of bonds between particles of aggregates and the cement medium which are not re-established.

According to Hadley,<sup>(144)</sup> concrete becomes fragmented at high loads and progressively loses the capacity to take load. The plastic

property shown by concrete is actually a metamorphosis of "concrete" into "discrete," where particles are held together not by cohesion, but by shear and aggregate interlock.

The onset of the internal rupture may be taken to be somewhere around half of the ultimate strength of concrete; the exact limit cannot be determined because it is a gradually developing phenomenon without any sharp point of onset. Even if some definite arbitrary criterion could be used to define the point of onset, this limit would be a function of several factors such as age of the specimen, moisture content, water-cement ratio, richness of concrete, type of aggregate, manner of compaction, etc.

Considering all of this, it appears that for stresses beyond about 50 per cent of the ultimate strength of concrete, what is measured as creep might include some deformation resulting from the internal rupture of concrete. It is interesting and perhaps significant to observe that this is about the limit above which creep strains tend to become non-linear with respect to the sustained stress. This also is approximately the fatigue strength of average concrete. The rapid increase in the apparent Poisson's ratio and onset of stethoscopically audible cracking noises reported by some investigators also occur at about this stress level.

## VII. THEORY OF RHEOLOGICAL MODELS

### A. GENERAL CONCEPTS

A body of matter, if subjected to a state of stress, is thereby placed in a state of strain. The response depends on the nature of the matter in question. The study of the relation between stresses and strains, in the most general sense, forms the subject matter of the branch of physics called rheology.

As a tool for the study of rheological properties of matter, theoretical ideal bodies with rigorously defined rheological properties (mainly elasticity, viscosity, and plasticity) have been postulated to facilitate the study and analysis of the behavior of real materials by comparing their properties with these idealized characteristics. The general approach consists of the following basic steps. A set of idealized "elementary" properties are defined and the theoretical rheological behavior of the associated "elementary bodies" or elements is deduced. To study a given material, the general deformation behavior is observed and a set of rheological measurements obtained from actual tests. A theoretical structural model is then postulated which contains various types of elements interacting in such a way as to result in a rheological behavior as close as possible to the observed behavior of the material under study.

The setting up of theoretical structural models is often not easy. Representation by mechanical models is often resorted to, to

facilitate their development. Such models provide a good qualitative guide for the study of rheological behavior and greatly help in the development of structural models. Behavior of mechanical models is relatively easy to visualize, and facilitates the setting up of mathematical equations so necessary for describing rheological behavior quantitatively. A mechanical model, however, should not be considered as representing the corresponding real material except for the rheological behavior.

### B. MODEL ELEMENTS

The basic elements of mechanical models usually consist of ideally elastic, ideally viscous, and ideally plastic mechanisms, represented respectively by springs, dash pots, and frictional elements.

#### 1. The Ideal Spring

A perfectly elastic body, usually called a Hooke solid, is one which exhibits the property of regaining its original shape and size completely and immediately as soon as the deforming stresses are removed. In the absence of any external load it has a definite shape and its deformation is uniquely defined by the stress system acting on it. This stress deformation relation may be linear or nonlinear. In the former case the body is called a Hookean body, in the latter case, a non-Hookean body. The body, when subjected to a homogeneous shearing stress  $\tau$ , will undergo a homogeneous shearing strain  $\gamma$  given by:

$\gamma = \frac{\tau}{G}$ , if the elasticity is linear  
and  $\gamma = F(\tau)$ , in the general case,  
 $\gamma$  and  $\tau$  being single valued functions of each  
other with  $\tau = 0$  for  $\gamma = 0$ .

In mechanical models this body is represented by an ideal spring, the extension  $x$  of which is a function of force  $p$  acting on it, and is completely and immediately recovered if the force is withdrawn. It is governed by the equation

$x = \alpha p$ , if the spring is linear

or  $x = F(p)$ , if the spring is nonlinear,  
 $x$  and  $p$  being single valued functions of each other, with  $x = 0$  for  $p = 0$ . The general relationship may be expressed as:  $x = \alpha p$ , where the compliance  $\alpha$  may be a function of either the displacement  $x$ , the force  $p$ , or both.

The elastic body is characterized by the facts that its rheological behavior is time-independent and that no energy is dissipated during a loading and unloading cycle.

## 2. The Ideal Dashpot

An ideally viscous body is that which undergoes a shearing deformation at a rate which is a function of the imposed shearing stress. If there exists a proportionality between the two, the body is said to be a linearly viscous body or a Newtonian liquid, otherwise a nonlinear or nonNewtonian liquid.

$\tau = \eta \dot{\gamma}$ , if it is linear

or  $\tau = F(\dot{\gamma})$ , otherwise.

Both the equations can also be written as

$\tau = \eta \dot{\gamma}$

where the coefficients of viscosity  $\eta$ , may be a function of one or more of the quantities, shearing stress  $\tau$ , shearing deformation  $\gamma$ , and time  $t$ . The dot over  $\dot{\gamma}$  represents its derivative with respect to time.

A viscous liquid is characterized by the fact that its deformation does not give rise

to any potential energy and dissipates all the work performed on it as heat energy, as against a perfectly elastic body which stores all the work done on it as potential energy.

In mechanical models, an ideal viscous body is represented by an ideal dashpot consisting of a piston moving through a viscous fluid of viscosity  $c$  contained in a container. When a load  $p$  is sustained on the piston, it moves with a velocity  $\frac{dx}{dt}$  which is a function of the applied load, the relation defining them being  $p = c \frac{dx}{dt}$ . The coefficient  $c$  may be a constant or a function of one or more of the quantities: time  $t$ ; displacement  $x$ ; or load  $p$ . If  $c$  is a constant or a function of  $t$  only, the dashpot is linear and represents a linearly viscous body. If  $c$  is a function of  $x$  or  $p$ , or both, the dashpot is said to be nonlinear.

A dashpot cannot store any potential energy and dissipates all the work done on it as heat energy. Positive work has thus to be done on it irrespective of the direction of deformation.

## 3. The Ideal Frictional Element

The ideal frictional element, or Saint Venant's body, is represented by a block resting on a flat surface. When a force is gradually applied to the body, the frictional force between the two surfaces in contact tends to oppose it. No movement ensues provided the force is smaller in magnitude than the force required to overcome the static frictional force between the surfaces. Once movement starts, a force equal to the kinetic friction only is sufficient to maintain the motion. The resistance is independent of the displacement or its time derivatives.

## C. ELEMENTARY MODELS

The ideal spring and the ideal dashpot may be employed as elements of mechanical

models to simulate rheological behavior of various materials. Sometimes, some additional fundamental behavior not represented by the spring and dashpot elements has to be incorporated to reproduce more adequately the behavior of a certain material. For example, frictional contacts to simulate materials with a yield point, dashpots with non-return valves, dashpots connected to a reservoir of viscous liquid maintained at a constant height, etc., have been employed in more complex models. Springs and dashpots are, however, by far the most commonly used elements.

There are two basic arrangements of rheological elements in models: in series and in parallel. When they are arranged in series, they take the same stress, the total displacement of the system being the sum of the individual displacements. When they are connected in parallel, they undergo the same displacement, and the total stress is the sum of the stresses in the individual elements. These principles are also applied in building complex bodies which may contain a number of general elastic and viscous elements in various series and parallel groupings. The knowledge of the fundamental properties of the elements and the rules which they obey when connected in series or parallel enables one to develop the equations of rheological behavior of a given model. The elements thus combined may be linear or nonlinear; the choice in any given case is guided by the observed rheological properties of the actual material and the knowledge of its structure. The solution of the equations of rheological behavior become laborious for even simple models if they are nonlinear.

If, in a given model, a section can be drawn which passes through springs only, the model will exhibit the characteristic of undergoing an instantaneous deformation on loading, which will be recovered instantaneously on

unloading. If a section can be passed through dashpots only, the model will exhibit an increasing deformation under a sustained load, which is essentially a property of liquids.

#### 1. The Maxwell Body

When an ideal spring is connected in series with an ideal dashpot, the system obtained is referred to as a Maxwell body (Figure 37-a). When a load  $p$  is suddenly applied to and sustained on this body, the spring undergoes an instantaneous extension  $x_s$  given by  $x_s = \frac{p}{k}$  where  $x_s$  = deformation of the spring and  $k$  = effective\* stiffness coefficient of the spring, on which will be superimposed a continuous deformation at a rate given by

$$\frac{dx_d}{dt} = \frac{p}{c},$$

where  $x_d$  = deformation of the dashpot. The total displacement  $x$  of the system at a given time will be given by the sum of the deformations of the spring and the dashpot.

The rheological equations thus would be:

$$x = x_s + x_d$$

$$p = kx_s = c \frac{dx_d}{dt}.$$

They may be combined to give the differential equation of the model,

$$\frac{\dot{p}}{k} + \frac{p}{c} = \dot{x}$$

which defines the relation between  $x$  and  $p$ ,  $\dot{x}$  and  $\dot{p}$  being their time derivatives.

The equation indicates that if the value of  $c$  is high, the part  $\frac{p}{c}$  can be neglected in comparison with  $\frac{\dot{p}}{k}$  and the equation reduces to  $\frac{\dot{p}}{k} = \dot{x}$  which gives, on integration, the equation of a spring. Thus for high viscosity of the dashpot, the Maxwell body degenerates into a Hookean body.

\* The term "effective" is used to include the general case of  $k$  varying with  $p$ ,  $x_s$  or  $t$ .

Similarly for a large value of  $k$ , or negligible rate of change of load, the first term becomes negligible and the equation reduces to that of a dashpot only. Thus the behavior of a Maxwell body, according to whether  $c$  or  $k$  is increased indefinitely, approaches that of a spring or a dashpot, respectively. Due to its tendency to undergo unlimited deformation under load, the Maxwell body is essentially a liquid.

If a load  $p$  is applied to a linear Maxwell body suddenly at  $t = 0$ , and sustained, the displacement as a function of time, will be as shown in Figure 37-b. If subsequently the load is taken off suddenly, the behavior will be given by BCD, BC being an instantaneous recovery equal in magnitude to OA but opposite in sense. The model will suffer a permanent deformation equal to CE which corresponds to the deformation undergone by the dashpot, and can be recovered only by a load applied in the opposite sense.

In a nonlinear Maxwell body any nonlinearity of the spring may show up as the inequality of the displacements OA and BC, and any nonlinearity in the dashpot may show up as a tendency for the line AB to be curved.

Under constant deformation a Maxwell body shows an important phenomenon called "relaxation of stress." In a linear Maxwell body the load relaxes exponentially. If in the equation for its response,  $\dot{x}$  is taken as zero and the equation is solved for  $p$ , we get

$$p = p_0 e^{-k/ct}$$

where  $p_0$  = initial load and  $e$  = base for natural logarithm. Theoretically, the stress relaxes completely as the time increases indefinitely (Figure 37-c). The equation  $p = p_0 e^{-k/ct}$  may also be written as  $p = p_0 e^{-t/\tau_M}$  where  $\tau_M = \frac{c}{k}$ , the quantity  $\tau_M$  is called the relaxation time of the model and

represents the time in which the stress relaxes to  $1/e$  times its original value. It is also the time in which the stress would relax to zero if it relaxes at the initial rate of relaxation.

## 2. The Kelvin Body or the Voigt Body

When an ideal spring is connected in parallel with an ideal dashpot, the resulting system is referred to as a Kelvin body or a Voigt body (Figure 38-a). The former name will be used here.

When a load  $p$  is suddenly applied and sustained on this body, the body does not show an instantaneous deformation as did the Maxwell body, but if the model elements are linear, it undergoes a deformation which increases with time exponentially. The deformations in both the elements are equal at any given instant and the load  $p$  is shared by both the elements depending upon the deformation and the rate of deformation of the system. The entire load is initially taken up by the dashpot, and as time goes on, it is transferred to the spring at a gradually decreasing rate until the entire load is transferred to the spring after an infinite time. The deformation of the system asymptotically approaches a value equal to the deformation which the spring alone would acquire under the load in the absence of the dashpot, i.e., a value  $p/k$  where  $k$  is the effective stiffness of the spring. The rheological equations neglecting the effects of inertia will be:

$$\begin{aligned} p &= kx_s + c\dot{x}_d, \\ \text{but } x &= x_s = x_d, \text{ therefore} \\ p &= kx + c\dot{x}. \end{aligned}$$

For the boundary condition  $x = 0$  at  $t = 0$ , the solution of this equation for linear elements will be:

$$x = \frac{p}{k} (1 - e^{-t/\tau_K}) \text{ where } \tau_K = \frac{c}{k}.$$

$\tau_K$  is the retardation time of the model and is a quantity representing the time taken by the model to attain a displacement equal to  $1/e$  of its ultimate magnitude. It is also the time that the model would have taken to acquire the ultimate deformation at a constant rate of deformation equal to the initial value.

If after time  $t_1$  after loading the load is removed, the displacement-time curve (Figure 38-b) will be OAC instead of OAB, the latter corresponding to a load sustained indefinitely. The displacement, after the removal of the load, decreases exponentially again to an asymptotic value equal to the initial zero value. This property of the Kelvin body is peculiar and is called delayed elasticity because after removal of the load the body finally returns, at least theoretically, to the initial state of deformation. The terms elastic aftereffect and anelasticity have also been used to describe this behavior. The Kelvin body is thus essentially an elastic solid with its elasticity rendered time-dependent due to the presence of a viscous body sharing its load.

The Kelvin body degenerates into a spring on one side and a dashpot on the other side, as limiting cases, according as the responses corresponding to the second or the first term of the differential equation become relatively negligible.

The general behavior, in case of non-linearity of the elements will be similar; the curve, however, may not be exponential. The slope of the curve increases as the viscosity increases and vice versa. The asymptote, too, may not remain a straight line.

### 3. The Hooke-St. Venant Body

The Hookean body and the St. Venant body, when coupled in series (Figure 39), represent a behavior which is typical of mild steel, for example. Materials having a definite yield

point can be represented by this body.

The St. Venant body may be generalized and combined in series with the Hookean body to simulate the behavior in the strain-hardening range that many engineering materials possess. A generalized St. Venant body is imagined as a number of St. Venant bodies arranged in series and connected to each other by flexible links. As the load is applied at one end of the system, movement starts as soon as the static friction of the body nearest to the load is overcome. Movement can be maintained at a uniform velocity by a force equal to the kinetic friction until the link connecting the first and the second body becomes taut. This body will start moving when its static friction is overcome, the process continuing until all the bodies come into play. To make the displacement time curve smooth, an infinite number of St. Venant bodies would be needed.

### 4. The Bingham Body

A body which exhibits a yield point beyond which it flows in a viscous manner is referred to as a Bingham body. The corresponding mechanical model is a series coupling of a spring, a St. Venant body, and a linear dashpot, with the free end of the dashpot fixed and the load being applied at the free end of the spring (Figure 40). If the friction limit for the St. Venant body is  $\theta$  (difference between kinetic and static friction may be neglected) and viscosity of the dashpot is  $c$ , the strain rate is given by

$$\dot{x} = 0 \text{ for } p \leq \theta$$

$$\dot{x} = \frac{p-\theta}{c} + \frac{\dot{p}}{k} \text{ for } p > \theta.$$

This model was formulated by Bingham, who used it to describe the behavior of paints. If the dashpot is not linear the body is called the generalized Bingham body.



## 5. The Burgers Body

If a Maxwell body is combined in series with a Kelvin body the resulting complex body is termed a Burgers body (Figure 41).

The total deformation of this body may be obtained, in case of linear elements, by superposition of the individual deformations of the Maxwell and the Kelvin parts under the same load.

If a load  $p$  is applied to and sustained on a Burgers body, an instantaneous deformation results followed by a time-dependent deformation increasing at a decreasing rate, the curve tending asymptotically to an inclined straight line instead of a horizontal straight line as in case of a Kelvin body. If the load is subsequently removed, an immediate recovery will follow, followed by a delayed recovery such that the recovery curve exponentially approaches a horizontal straight line. A part of the deformation is not recovered unless a load in the opposite sense is applied.

These various components of deformations may be identified as belonging to different elements of the model. The initial instantaneous deformation is due to the spring of the Maxwell body. The slope of the asymptote is due to the viscous deformation in the dashpot of the Maxwell body. The intercept EA on the deformation axis is the deformation that would be attained by the Kelvin body for the load sustained indefinitely. The exponential part of the deformation on loading and subsequent decay on unloading corresponds to the behavior of the Kelvin body. The irrecoverable deformation corresponds to the total deformation in the Maxwell body produced during the time the load was sustained. It may be noted here that these components are strikingly similar to the components of a typical creep curve for concrete. This suggests the use of the Burgers model as a possible representation of concrete,

at least as far as its creep behavior is concerned. This will be discussed in detail later.

The rheological equations for the Burgers body with linear coefficients are:

$$p = k_M x_{sM} = c_M \dot{x}_{dM} = k_K x_K + c_K \dot{x}_K$$

and

$$x = x_{sM} + x_{dM} + x_K$$

or

$$x = \frac{p}{k_M} + \frac{p t}{c_M} + \frac{p}{k_K} (1 - e^{-t/\tau_K}),$$

where  $\tau_K = \frac{c_K}{k_K}$ , the retardation time of the Kelvins body

$$= p \left[ \frac{1}{k_M} + \frac{t}{c_M} + \frac{1}{k_K} (1 - e^{-t/\tau_K}) \right]$$

where  $x_{sM}$ ,  $x_{dM}$ , and  $x_K$  are the displacement components of the Maxwell spring, Maxwell dashpot, and the Kelvin body respectively.

The Burgers body shows a property of stress relaxation like that of a Maxwell body. For strain maintained constantly at a value  $x$ , the load relaxes according to the following expression:

$$p = \frac{c_M k_K x}{c_M + k_K} e^{-\left(\frac{k_K t}{c_M + k_K}\right)}.$$

If a load sustained on a Burgers body is not constant with time the deformation is given by the differential equation,

$$\ddot{x} + \frac{k_K}{c_K} \dot{x} = \frac{p}{k_M} + \left[ \frac{1}{c_K} + \frac{1}{c_M} + \frac{k_K}{c_K k_M} + \frac{k_K}{c_M c_K} \right] x.$$

The Burgers body shows an interesting effect called the "memory effect."<sup>(140)</sup> If for example the system were subjected to a load  $p$ , the system will, as stated before, undergo an instantaneous deformation followed by a deformation gradually increasing at a decreasing rate acquiring a limiting rate as time increases indefinitely. Now, if, after



the deformation rate is close to its limiting value, the load is suddenly removed, it will be followed by an immediate recovery of deformation due to the Maxwell spring, followed by a delayed recovery of the Kelvin's body. If after a small time a force in the opposite direction were applied, it will be followed by an instantaneous reduction in the total deformation due to the Maxwell spring. If the reversed load is removed after a short time, the Maxwell spring extension will instantaneously follow. This short duration loading of the system will not significantly influence the tension in the Kelvin spring. Now, under no load again, the system will tend to contract at a gradually diminishing rate, reaching asymptotically a value, which, however, will be a little lower by the amount the Maxwell dashpot had been deformed during the time the Maxwell spring was loaded with a reversed force. This characteristic of the Burgers body of remembering the load history is called the "memory effect."

#### 6. The Double Maxwell Body

If two Maxwell bodies are connected in series, the net behavior of the combined body is equivalent to that of a single Maxwell body. When subjected to a constant load it shows an instantaneous deformation equal to the sum of the deformation of the two springs, followed by a viscous deformation at a rate which is the sum of the rates of deformations of the individual dashpots. If two identical Maxwell bodies are connected in parallel, the net behavior is again the same as that of a single Maxwell body, except that for the same load, the instantaneous deformation and the rate of subsequent deformation will be half the corresponding quantities in either of the Maxwell bodies isolated and subjected to the same load.

If the two Maxwell bodies with unequal

relaxation times are connected in parallel, the behavior of the combined body resembles that of a Burgers body. It can be seen that as a section can be passed wholly through the two springs the body will show an instantaneous elastic response. The model in fact shows a net stiffness equal to the sum of the stiffnesses of the individual springs. Similarly it can be observed that the body is essentially a liquid and for linear elements must eventually exhibit a constant rate of deformation, its net viscosity being the sum of the component viscosities. The model, however, also shows a behavior of delayed elasticity due to the gradual decrease in the rate of deformation caused by relaxation of the individual springs. The actual initial rate of deformation of the body will depend on the relative magnitudes of the characteristic values of the various elements. It can be seen that this model is similar to the Burgers body. In fact, they can be shown to be governed by the same form of differential equations and are thus mathematically equivalent. If the two bodies shown in Figures 41 and 42 are identical in their behavior, the following relations must hold good.

$$\begin{aligned}
 k_M &= k_M' + k_M'' \\
 c_M &= c_M' + c_M'' \\
 k_K &= \frac{k_M' k_M'' (c_M' + c_M'')^2 (k_M' + k_M'')}{(c_M' k_M'' - c_M'' k_M')^2} \\
 c_K &= \frac{c_M' c_M'' (k_M' + k_M'')^2 (c_M' + c_M'')}{(c_M' k_M'' - c_M'' k_M')^2}
 \end{aligned}$$

A reverse transformation is also possible but the expressions involved are more complex.

Since the double Maxwell model is very indirect in its behavior under constant stress, the equivalent Burgers body is usually preferred. However, the former is more suitable for obtaining stress-strain curves at constant

rates of deformations.

#### D. GENERALIZED REPRESENTATION

It may not always be possible to explain the behavior of real viscoelastic material by simple mechanical models. Increasingly complex models with linear elements cannot describe the behavior, and simple or complex models with nonlinear elements have to be used for proper simulation of the behavior in question.

Any model composed of springs and dashpots can be set up and by sufficiently increasing the complexity, a suitable model can be devised to describe a linear behavior. However, such complex models can always be reduced to one of the four fundamental or "canonic" forms. If a spring is regarded as a limiting case of either a Maxwell body with infinite viscosity or a Kelvin body with zero viscosity and if a dashpot is regarded as a limiting case of a Maxwell body with an infinite spring stiffness or a Kelvin body with a zero spring stiffness, it can be said that all the four canonic forms follow from the two fundamental models: the general Maxwell model and the general Kelvin model. The former is a parallel combination of an arbitrary number of Maxwell units, while the latter is a series combination of an arbitrary number of Kelvin units. The two general bodies have to have an identical number of total elements for correspondence.

Reiner<sup>(49)</sup> has used a shorthand for writing, in formula forms, the structure of various models. Using a characteristic letter for an element or a combined body, a dash for a series connection, and a vertical bar for a parallel connection, a Maxwell body can be described as  $M = H-N$  which reads, "a Maxwell body is a Hookean body connected in series with a Newtonian body." Similarly, a Kelvin body would be represented by  $K = H|N$ , a Burgers body by  $B_u = H-N-(H|N)$  or  $M-K$ . This

method may be extended to describe a general Maxwell body as  $(|M|)^n$  and a general Kelvin body as  $(-K)^n$  where  $n$  is any arbitrary number. The structural formulas for the four canonic forms, in two alternative forms become:

$$C_1 = H - (-K)^{n-1} = H (|M|)^{n-1}$$

$$C_2 = H-N-(-K)^{n-2} = (|M|)^n$$

$$C_3 = N-(-K)^{n-1} = N(|M|)^{n-1}$$

$$C_4 = (-K)^n = H|N(|M|)^{n-2}.$$

The sketches of these models are easy to construct from these formulas.

Considering a Hookean body as a Maxwell body with an infinite relaxation time or a Kelvin body with a zero retardation time; and a Newtonian body as a Maxwell body with zero relaxation time or a Kelvin body with an infinite retardation time, each of the canonic forms may be recognized as having  $n$  different relaxation or retardation times. As the real material to be represented becomes more and more complex in mechanical behavior, more and more elements become necessary and  $n$  approaches infinity. A model like this can be said to possess an infinite number of relaxation or retardation times of varying magnitudes. Diagrams defining these relative magnitudes for different materials are called relaxation or retardation spectra.

In practice, such a complicated model is difficult to analyze. It is usual to approximate it by a simplified setup which would give a reasonably similar response. Consider, for example, a real material the mechanical model of which is represented by a number of Kelvin bodies with different retardation times, in series. To simplify this, all these Kelvin bodies may be grouped into three divisions according to their retardation times. Those with the smallest retardation times may be

approximated by a spring and those with largest retardation times may be approximated by a dashpot only. The remaining Kelvin bodies can then be approximated by a single Kelvin body, the retardation time of which is such that the resultant load displacement curve is closest to that given by the original model. Thus we obtain what is called a four parameter model. If the agreement between the behavior of the simplified model and the original model is not close, all the Kelvin bodies in the original model can be broken up into a larger number of groups. Each additional group contributes two more parameters to the simplified model, thus increasing its complexity.

#### E. THE PRINCIPLE OF SUPERPOSITION

Boltzmann's principle of superposition plays a very important part in the theory of viscoelasticity. It was derived by Boltzmann (145) from empirical considerations and studied in detail by Leaderman.<sup>(146)</sup> Essentially it states that if a series of loads are applied to a linearly viscoelastic body at different times, the deformation at any given time is the sum of all the deformations that the body would have undergone at that time, had each load been applied individually at its corresponding time. Thus, if the application of force  $p$  on a viscoelastic body produces a deformation  $x_t$ , after time  $t$ , governed by the expression

$$x_t = p \left[ \frac{1}{k} + C_c F_c(t) \right]$$

(where  $k$  and  $C_c$  are constants and  $F_c(t)$  is the creep function) then the total deformation  $x_T$  of the body at any time  $T$  after application of forces  $p_i$ ,  $i = 1, 2, 3, \dots, n$ , at times  $T_i$  and sustained thereafter will be given by the following expression if the body obeys the principle of superposition:

$$x_T = \sum_{i=1}^n p_i \left[ \frac{1}{k} + C_c F_c(T - T_i) \right].$$

Similarly if the load is given as a function of the time  $t$  after application of a deformation  $x$  by the expression

$$p_t = x \left[ k + C_r F_r(t) \right]$$

(where  $k$  and  $C_r$  are constants and  $F_r(t)$  is the relaxation function) then for different deformations  $x_i$  imposed on the body at times  $T_i$ , and sustained, the load at time  $T$  is given by

$$p(T) = \sum_{i=1}^n x_i \left[ k + C_r F_r(T - T_i) \right].$$

The summation in a continuous case can be replaced by integration.

This principle is obeyed by all materials whose rheological behaviors are governed by linear differential equations with constant coefficients, and whose structures are time-independent.

#### F. VIBRATIONAL RESPONSE OF RHEOLOGICAL MODELS

In the mechanical behavior of viscoelastic materials the general problem is to find, for any sequence of homogeneous shear stresses, the strain response as a function of time, or, for a sequence of enforced homogeneous strains, to find the stresses set up. There are certain cases of special interest, namely: response to constant stress or constant load (i.e., creep), response to constant deformation (i.e., relaxation), stress response at a constant strain rate, and stress or strain variation with time for sinusoidal variation of applied strain or stress respectively (i.e., the dynamic response). It was pointed out earlier how the first two kinds of behavior can be studied with the aid of rheological models. The dynamic response of viscoelastic

materials can likewise be studied by analyzing the corresponding response of analogous mechanical models.

It may be observed that while studying the creep and relaxation responses of the models, the acceleration of the mass of the system was ignored under the assumption that the inertia forces were negligible in comparison with other forces involved. This, however, cannot be justified in the study of dynamic behavior. As a matter of fact, the inertial effect itself becomes the main object of study. In real viscoelastic materials the mass would be uniformly distributed, but in the corresponding mechanical analogs, the mass is assumed as concentrated at one or more points. In systems of all but the simplest types, the problem of the distribution of the mass between various component systems becomes exceedingly complicated.

#### 1. Response of Elements

If an ideal linear spring of stiffness  $k$  is connected at one end to a rigid support and a mass suspended from the other end of it, we get the simplest system capable of vibrating. If the mass is given a small displacement and then released, the system will exhibit a vibratory motion. The displacement will be a sinusoidal function of time, the motion being called a simple harmonic motion. It is governed by the differential equation

$$m\ddot{x} + kx = 0^*$$

the general solution of which can be written as  $x = A \sin \omega t + B \cos \omega t$  where  $A$  and  $B$  are arbitrary constants determined by the initial conditions in the particular case, and  $\omega$  is the angular frequency of the system given by  $\omega = k/m$ . The natural frequency of the system is given by  $f = \omega/2\pi$  and represents the number

of complete cycles of motion that the system will undergo in a unit time. The natural frequency (or its reciprocal, the period) is one of the most important parameters of a vibrating system.

If two springs of stiffnesses  $k_1$  and  $k_2$  are connected in series, the resultant effective compliance (for the purpose of vibrational response) is given by the sum of the compliances of the individual springs. If, however, the two springs are connected in parallel so that they undergo equal deformation, the effective stiffness is the sum of the individual stiffnesses.

An ideal dashpot, if subjected to an initial deformation away from its equilibrium position, will stay there unless moved by some other external force. The concept of free vibrations of a dashpot by itself thus has no meaning. However, dashpots play a significant role in vibrational response, as they are responsible for dissipation of energy stored in the spring elements. If in a system with dashpots no external energy is supplied the vibratory motion of the system if started gradually comes to a stop. Maintenance of the vibratory motion at a constant amplitude thus requires replenishment of energy. The amount of replenishment needed is equal to that dissipated in the dashpots as heat. The former is called a system with damped free vibrations.

#### 2. Response of a Kelvin Body

If a linear Kelvin body with a spring stiffness  $k$ , a dashpot coefficient  $c$ , and a concentrated mass  $m$ , is given an initial deformation and released, its motion will be governed by the differential equation  $m\ddot{x} + c\dot{x} + kx = 0$ . The system will undergo damped free vibrations, provided the coefficient  $c$  is less than a certain value  $c_c$  (called the critical damping coefficient and given by

\* For derivation of the formulas in this section, refer to any standard textbook on vibrations.

$c_c = 2\sqrt{km}$ ).  $\frac{c}{c_c}$  is usually referred to as the damping factor  $\zeta$ . For  $\zeta$  less than unity, the solution for the above differential equation is given by

$$x = e^{-\zeta\omega_n t} \left[ \alpha \sin \sqrt{1-\zeta^2}\omega_n t + \beta \cos \sqrt{1-\zeta^2}\omega_n t \right]$$

where  $\alpha$  and  $\beta$  are arbitrary constants, and  $\omega_n = \sqrt{\frac{k}{m}}$  (i.e., the undamped natural frequency). The successive maximum amplitudes have a practically constant ratio and the envelope of the displacement-time diagram is a pair of exponential curves (Figure 43). The natural logarithm of the constant ratio of successive maximum amplitudes is called the logarithmic decrement of the system and is given by the expressions

$$\delta = \ln \frac{A_n}{A_{n+1}} = \frac{1}{n} \ln \frac{A_0}{A_n} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$A_n$  being the amplitude after  $n$  cycles. For small values of  $\zeta$ ,  $\delta$  may be approximated by  $2\pi\zeta$ . The logarithmic decrement also corresponds (for small values of the damping factor  $\zeta$ ) to one half the ratio of the energy dissipated per cycle to the total vibrational energy of the system (i.e.,  $\delta \cong \frac{\Delta W}{2W}$ ). Thus  $\delta$  is a convenient measure of the dissipation of energy of a viscously damped system for small damping factors.

If the same Kelvin body is subjected to a force, varying sinusoidally with time, a state of forced vibrations ensues. The motion is then governed by the differential equation

$$m\ddot{x} + c\dot{x} + kx = p_0 \sin \omega t,$$

where  $p_0$  is the maximum value of the imposed force.

The solution of the equation is given by

$$x = A e^{-\zeta\omega_n t} \sin (\sqrt{1-\zeta^2}\omega_n t + \phi_1) + \frac{p_0 \sin(\omega t - \phi)}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}.$$

The first part of the expression is called the

transient term and the second part the steady-state solution. As may be seen, the first part will be damped out in a short time and the motion will subsequently be governed by the second part only. The maximum steady-state amplitude

$$x = \frac{p_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}, \text{ which}$$

may be put into the following very convenient nondimensional form

$$X = \frac{X_0}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}},$$

where  $X_0 = p_0/k$  = the deflection of spring for the force  $p_0$ , applied statically, and  $r = \omega/\omega_n$ . The angle of lag  $\phi$  by which the displacement lags the impressed force is given by

$$\phi = \tan^{-1} \frac{2\zeta r}{1 - r^2}$$

It may be observed that for a given  $\zeta$ , the ratio  $X/X_0$  (called the magnification factor) is a function of  $r$ . A curve of  $X/X_0$  versus  $r$  can be drawn for any given value of  $\zeta$ . A set of curves for different values of  $\zeta$  is shown in Figure 44.

For a given  $\zeta$ , the curve is characterized by the fact that  $X/X_0$  is equal to unity for a zero value of  $r$  and increases continuously, initially at a slow rate then rapidly reaches a maximum for a value of  $r$  somewhat less than unity. As the value of  $r$  is further increased,  $X/X_0$  decreases at a decreasing rate as  $r$  increases indefinitely. The angular frequency  $\omega$ , for which the value  $X/X_0$  reaches a maximum, is called the resonant angular frequency. For small values of  $\zeta$ , this occurs when:  $r = 1$ , the displacement lags the impressed force by about  $90^\circ$ , and the full impressed force almost completely counterbalances the damping force.

As the value of  $\zeta$  changes, although the

general nature of the curves remain the same,\* there is a marked influence in the vicinity of  $r = 1$ . As  $\zeta$  is decreased, the maximum value of  $X/X_0$  increases rapidly, and becomes theoretically infinite for  $\zeta = 0$ . The curves tend to show a smaller and smaller spread as  $\zeta$  decreases. The spread of the resonance curve serves as a convenient measure of  $\zeta$  and forms the basis of obtaining damping capacity of materials by resonance techniques. If two values of  $r$ , and hence  $\omega$ , are obtained (for which the magnification factor is  $\frac{1}{n}$ th part of its maximum value), the damping factor  $\zeta$  can be obtained, for small values of  $\zeta$ , from the expression

$$\zeta = \frac{\Delta \omega}{2\omega_n \sqrt{n^2 - 1}},$$

where  $\Delta \omega$  is the difference in the two values of  $\omega$  for a given  $n$ . For  $n = 2$  the expression reduces to

$$\zeta = \frac{\Delta \omega}{2\omega_n}.$$

From this (the value of  $\zeta$ ), the logarithmic decrement  $\delta$  can be obtained by means of the expression  $\delta = 2\pi\zeta = \pi \frac{\Delta \omega}{\omega_n}$ .

### 3. Response of a Maxwell Body

A Maxwell body, when subjected to a sinusoidal force  $p_0 \sin \omega t$ , constitutes a system with two degrees of freedom (Figure 45). A general solution of this system thus becomes quite difficult. However, its limiting cases can be studied qualitatively. When the value of  $\omega$  is very high, the dashpot acts as a rigid body and the behavior of the system is like that of a Hookean body with stiffness  $k$ . On the other hand, if the value of  $\omega$  is very low, the spring never gets a chance to store energy

and remains relaxed, and the linear system will be governed by the equation:

$$x \cong \frac{\sin(\omega t - \phi)}{(\mu\omega^2)^2 + (\alpha\omega)^2}$$

where

$$\phi = \tan^{-1} \frac{c}{\mu\omega}.$$

A Maxwell body dissipates energy at a rate of  $\pi p_0^2 / \omega$  where  $p_0$  is the maximum value of the force and  $\omega$  its frequency.

### 4. Response of Models with Variable Elements

In a general case the elements may possess properties which vary with time, load, and displacement. The solution of the resulting differential equations governing the static or dynamic behavior of the model can become much more complex than when invariable elements are involved. In most cases rigorous solutions are not even available and recourse has to be taken to approximate methods or analog techniques.

The problem is more tractable when the elements are only time dependent, in which case the behavior can be represented by linear differential equations with variable coefficients. Closed-form solutions are available for several such types. Flügge<sup>(55)</sup> has presented solutions for the displacement of a number of rheological models composed of springs and dashpots with time variable coefficients. Linear and quadratic time functions have been considered.

If, however, the properties of the elements also depend on the load and displacement, the governing differential equations are no longer linear. A very limited number of closed-form solutions are available for certain idealized cases; even these are exceedingly involved. In practice, the only possible approach is through approximate procedures or experimental techniques.

\*For values of  $\zeta$  greater than 0.5, even the shape becomes markedly different.



Mechanical Quantity		Electrical Quantity	
		Force-Voltage Analogy	Force-Current Analogy
Force	p	Voltage . . . . . e	Current . . . . . i
Velocity	v	Current . . . . . i	Voltage . . . . . e
Displacement $x = \int_0^t v dt$		Charge, $q = \int_0^t i dt$	---
Spring compliance	1/k	Capacitance . . . . C	Inductance. . . . . L
Damping coefficient	c	Resistance. . . . . R	Conductance . . . 1/R
Mass	m	Inductance. . . . . L	Capacitance . . . . C

#### G. THE ELECTROMECHANICAL ANALOGY

A very powerful and convenient tool for the study of the response of rheological models is provided by the analogy between electrical and mechanical systems whose behaviors are governed by the same type of differential equation. Both static and dynamic response can be very conveniently studied.

Two alternative analogies are available for representing the mechanical quantities in terms of their electrical counterparts, i.e., the Force-Voltage analogy and the Force-Current analogy. The corresponding analogous mechanical and electrical quantities for the two analogies are tabulated.

The Force-Voltage analogy is more commonly used. Figures 46 and 47 show the analogous elements, and circuits for the Maxwell,

Kelvin and Burgers bodies, under dynamic conditions. Replacing the time-dependent electromotive force by a constant voltage converts the circuits for representation of response under sustained loads. An integrating circuit is used to convert the measured currents to the corresponding charge or displacement analog.

Kennedy<sup>(147)</sup> has developed analogous electrical circuits representing the creep and recovery behavior of several real materials.

The electrical analogy is not limited to linear systems. With a suitable choice of nonlinear electrical elements, nonlinear viscoelastic response can be readily represented. Matching the nonlinear behavior of the corresponding electrical and mechanical elements may, however, pose a serious practical problem.



## VIII. RHEOLOGICAL MODELS FOR CONCRETE

### A. CONCRETE AS A VISCOELASTIC MATERIAL

Several authors, Thomas,<sup>(24)</sup> Reiner,<sup>(49)</sup> and Freudenthal<sup>(73, 105)</sup> among others, have described concrete as a viscoelastic material. Strong reasons exist, as discussed earlier, to believe that this is essentially so.

Creep and relaxation properties exhibited by concrete can be explained reasonably well by considering concrete to be a viscoelastic material. The mechanical behavior of concrete under constant load or constant deformation resembles, as a first approximation, that of a viscoelastic material. The instantaneous deformation (Figure 26) following the load is an elastic characteristic. The gradual increase in strain at a decreasing rate is a characteristic of a viscoelastic body composed of a viscous and an elastic part so arranged that, when under load, they have the same deformation. They share the imposed load in a proportion depending upon the rate of strain of the viscous part. The nearly steady rate of creep exhibited after a certain period is typical of a viscous body. The instantaneous recovery on release of load is again an elastic property which is followed by a time-dependent deformation called delayed elasticity, which is also a property commonly found in viscoelastic materials. Finally, the residual deformation after the delayed recovery is complete is a property exhibited by viscous bodies. The last property is also exhibited

by plastic bodies, but the facts that plastic bodies are susceptible to a permanent deformation when subjected to an impact loading without undergoing fracture, whereas viscous bodies are not, and that viscoelastic bodies may undergo a brittle fracture seems to indicate that concrete is a viscoelastic material rather than a plastic material since it exhibits a brittle fracture under impact loads exceeding its strength.

Reiner<sup>(49)</sup> considered hardened cement as a liquid on account of its property to flow with time, at least up to five or six years. In 1932, Bingham and Reiner<sup>(23)</sup> carried out an investigation to find out whether the aggregate itself flows during the creep of cement mortar. They observed the rate of creep in its approximately constant state to be the same irrespective of the previous history of the material, and found it to be greater for neat cement paste than for mortar. In a further investigation with Arnstein,<sup>(42)</sup> Reiner found proportionality of creep in paste and mortar, thus emphasizing the viscous character of the flow. These, along with the findings of Glanville and Thomas,<sup>(35)</sup> lead Reiner to believe that concrete and mortar could be considered as comprising two phases, the cementitious material behaving as a viscous fluid under load, and the inert aggregate which does not flow, but merely moves, under load.

Reiner postulated that the addition of

aggregate increases the viscosity of cement paste, which is responsible for the lower values obtained for creep in mortar. He further assumed that the increase in viscosity is governed by Einstein's equation, which states that in a dispersed liquid-solid system, where a solid is dispersed in a liquid, the apparent viscosity of the liquid is increased by a factor which is greater than unity by 2.5 times the volumetric concentration of the dispersed phase. He found that for cement mortars this relation held up to 60 per cent volume concentration of aggregate in the mortar. He presented the following formula for predicting the creep viscosity  $\eta_c$  of a mortar from the creep viscosity  $\eta_o$  of a neat cement paste of given composition:

$$\eta_c = \eta_o \frac{1 + \overline{WC}}{1 + WC} (1 + 2.5 v_2),$$

where WC and  $\overline{WC}$  are the water-cement ratios (by volume) of the neat cement paste; that used in the mortar and  $v_2$  is the volumetric concentration of the aggregate in the mix. For concrete he found the factor in the parenthesis to be

$$1 + 40v_2 - 53v_2^2.$$

The findings by Reiner thus provide a strong support in favor of considering concrete as a viscoelastic body.

Freudenthal<sup>(56)</sup> also describes concrete as composed of loose aggregate grains held together by a highly viscous liquid - the cement paste - the viscosity of which increases with time. The relative volumes of aggregate and paste, according to him, determine the mechanical behavior of concrete. The aggregate offers resistance to shear by virtue of the internal friction between the individual grains, making it predominate in lean concretes. The behavior of richer concretes is essentially

viscoelastic. He considers creep of concrete to be basically a viscoelastic phenomenon, its gradually decreasing rate being due to the increasing viscosity of the paste with time. Creep would thus stop either when a crystal-line network in the cement gel blocks the deformation or when the aggregate particles touch one another as a result of the creep in paste, forming a rigid skeleton, which would occur earlier in concretes with higher concentration of aggregates.

Hansen,<sup>(79)</sup> using a Burgers body as a rheological model, developed an expression for the basic creep of concrete which was found to check closely with available experimental data.

The above considerations support the view that concrete may be considered as a basically viscoelastic material. The view has been reinforced by recent findings on the dynamic behavior of hardened concrete. Specimens of concrete, when excited to vibration by an impulse, exhibit amplitude decay curves characteristic of viscous damping. (148, 149, 204)

A number of mechanical models based on the viscoelastic concept have been developed by various authors to simulate the deformation and strength characteristics of concrete. Notable among these are those proposed by Ross, Flügge, Cowan, Freudenthal and Roll, Torroja and Paez, Hansen, Chang and Kesler, Glücklich, and others. These will be briefly reviewed.

#### B. ROSS'S MODEL

The model proposed by Ross<sup>(40)</sup> seems to be the first one ever presented to describe the mechanical behavior of concrete. The model consists of a container with a porous piston and two springs with compliances  $\alpha_e$  and  $\alpha_c$  as shown in Figure 16, the container being partly full of a liquid of fluidity  $\Phi$ . Although apparently different in form, it is

actually a Kelvin body with a spring in series or a truncated Burgers body, H - K or H - (H N). If a stress  $\sigma$  is rapidly applied and subsequently sustained, there is an instantaneous deformation  $\epsilon_c = \sigma \alpha_e$  of the first spring followed by a progressive deformation of the second spring with time. The second spring gradually takes up an increasing load, until eventually a static condition is reached, the total deformation being  $\sigma(\alpha_e + \alpha_c)$ . The total deformation at any time  $t$  after application of the stress will be given by:

$$\epsilon = \sigma \left[ \alpha_e + \alpha_c \left( 1 - e^{-\frac{\phi t}{\alpha_c}} \right) \right].$$

The model also exhibits an instantaneous recovery  $\sigma \alpha_e$  on removal of the load, followed by a gradual recovery. To simulate irrecoverable deformation, an opposing stress  $q$  is assumed to be acting on the piston during the reversed motion. The recoverable strain  $\epsilon_r$  at any time  $t$  after application of the stress and its subsequent removal at time  $t_1$ , is given by:

$$\epsilon_r = \alpha_c \left[ (\sigma - q) e^{-\frac{\phi(t-t_1)}{\alpha_c}} - \sigma e^{-\frac{\phi t}{\alpha_c}} + q \right].$$

An internal stress  $s$  is assumed to act on the piston to represent shrinkage. Thus the shrinkage stress at time  $t$  is given by:

$$\epsilon_s = s \alpha_c \left( 1 - e^{-\frac{\phi t}{\alpha_c}} \right).$$

The total deformation after a time  $t$  would thus be

$$\epsilon = \sigma \alpha_e + (\sigma + s) \alpha_c \left( 1 - e^{-\frac{\phi t}{\alpha_c}} \right).$$

Average values of the constants for a particular concrete were found to be:

$$\alpha_e = 0.25 \times 10^{-6},$$

giving  $E_c = 4.0 \times 10^8$  lbs/sq.in.

$$\alpha_c = 0.5 \times 10^{-6} = \text{the ultimate creep per unit stress}$$

$$s = 400 \text{ psi}$$

$$\phi = 1.0 \times 10^{-6} \text{ when } t \text{ is expressed in years.}$$

The model, although satisfactory as a first approximation, is not very accurate as it does not take into account several fundamental characteristics of rheological behavior of concrete. For example, the model proposed cannot explain the phenomenon of relaxation of stresses as it is essentially a solid. Relaxation of stresses in concrete is an observed fact and a rational model should simulate it.

The model also predicts an exponential creep-time curve for concrete which is contrary to observation.

#### C. FLÜGGE'S MODELS

Flügge<sup>(55)</sup> analyzed creep from tests conducted by Davis and Davis<sup>(20, 93)</sup> and Glanville,<sup>(21)</sup> and developed models simulating the creep behavior of concrete. He reckoned zero time from the day when the concrete was made.

The model he used to describe the tests by Davis and Davis is shown in Figure 48. It consists of two dashpots arranged in series with a Kelvin body. One of the dashpots in series has a constant coefficient of viscosity while the other has a coefficient varying as the square of time. The dashpot of the Kelvin part varies linearly with time. The spring has a constant compliance  $\alpha_k$ . The components, two dashpots, and a Kelvin body are marked 1, 2, and 3, on the figure. Only time-dependent deformation is accounted for.

Under a constant stress  $\sigma$ , the strains of the elements 2 and 3 ultimately attain finite values, respectively equal to  $\sigma/\beta k$  and  $p\alpha_k$  where  $k$  is the age of loading. The deformation of the element 1 continues to increase

indefinitely at a constant rate. The coefficient of the first element can be calculated from the asymptotic strain rate,  $\sigma/c_1$ , while the coefficients for the elements 2 and 3 can be obtained from the fact that the limiting deformation for element 2 depends on the age at loading, while that of 3 does not.

For analyzing Glanville's tests, he used essentially the same model but, in case of the first of the three series analyzed, a Kelvin dashpot having a constant viscosity was used instead of one increasing linearly with time. For the second series he changed the dashpot of element 2 by making its viscosity proportional to  $(t + t_0)^2$ ,  $t_0$  being an additive constant.

The curves obtained for the creep response of the model agreed fairly well with the actual data for most of the curves studied. The model constants were found to vary widely with type of concrete, storage conditions, and age of loading.

#### D. COWAN'S MODEL

The model suggested by Cowan<sup>(127)</sup> consists of two Kelvin bodies in series with one of the Kelvin dashpots having a nonreturn valve (Figure 49). A set of brittle springs arranged in parallel is connected with it in series. The individual springs have different ultimate strengths. The model was devised by him not to explain time-dependent deformation but to explain the stress-strain response of concrete at a constant strain rate.

On application of a moderate load, an instantaneous elastic deformation will be observed due to the springs. This will be followed by gradually increasing deformation due to the two Kelvin bodies which will, ultimately, be proportional to the applied load. On unloading, the procedure is reversed, but a part of the deformation is not recovered due to the presence of nonreturn valves in one of the Kelvin dashpots. For rapid loading, there will be only negligible deformation in the

Kelvin bodies and the net response will be essentially elastic until the fracturing load of the weakest brittle spring is reached. When subjected to slow and heavy deformation both types of inelastic action -- that due to Kelvin bodies, partly irreversible, and that due to the rupture of springs, wholly irrecoverable -- would occur (Figure 50).

For rapid deformation rates at which the dashpots do not respond to the load, the model shows a stress-deformation curve which is very similar to an actual stress-strain curve for concrete. Initially the curve rises steeply and almost linearly, the slope decreasing gradually to zero at an increasing rate until the curve reaches a maximum which corresponds to the ultimate strength. As the deformation increases further, the stress decreases slowly at first, and at a fairly constant rapid rate thereafter until final fracture. The model describes fairly well the mechanical behavior of concrete as observed in a stress-strain relation during a test to destruction at a constant rate of strain.

#### E. FREUDENTHAL'S MODEL

In recognition that creep is a complex phenomenon caused by a number of phenomena, this model<sup>(73)</sup> was designed to take into separate account the deformations due to various causes. It consists of four components, each of which is composed of two elements. Of the four components, one is a Maxwell body and other three are Kelvin bodies, all connected in series. Each component is introduced to account for deformation resulting from a particular phenomenon. Several of the eight elements are nonlinear. The detailed model is shown in Figure 19. The model, in detail, consists of the following bodies combined in series:

1. A Maxwell body with a linear spring and a dashpot nonlinear with respect to force and time, representing the long time viscoelastic response in shear resulting in

irrecoverable creep;

2. A Kelvin body, with a linear spring and a linear dashpot, representing the viscoelastic interaction in shear of solid and fluid phases resulting in recoverable creep or delayed elasticity;

3. A Kelvin body, nonlinear with respect to force, representing the short-time consolidation effects due to irrecoverable pore-water motion (seepage) towards the surface and subsequent evaporation;

4. A Kelvin body, nonlinear with respect to force, representing the irrecoverable deformation due to partially destructive internal readjustments within the granular mass of the concrete.

Components 1 and 2 respond to both increasing and decreasing forces, while 3 and 4 respond to increasing forces only.

The creep strain  $\epsilon_c$  for a sustained stress  $\sigma$  at any time  $t$  after its application will be given by:

$$\epsilon_c = C e^{-b} \tau_M (1 - e^{-t/\tau_M}) + \sigma \alpha_K (1 - e^{-t/\tau_K}) + \sigma \alpha_1 (1 - e^{-t/\tau_1}) + \sigma \alpha_2 (1 - e^{-t/\tau_2})$$

where  $C = \text{unity}$ ,  $\tau_M$  is the relaxation time for the Maxwell body,  $b$  is an empirical constant, and  $\tau_K$ ,  $\tau_1$ ,  $\tau_2$  are the retardation times for the Kelvin bodies. To simplify the evaluation of the six Kelvin coefficients, it was considered reasonable to make the following assumptions:

$$\tau_1 = 10\tau_2 = \tau_K$$

$$\alpha_1 = \alpha_2 + \alpha_K$$

which reduce to four the total number of coefficients to be evaluated. The creep strain can now be expressed as:

$$\epsilon_c = C e^{-b} \tau_M (1 - e^{-t/\tau_M}) + \sigma \alpha_1 (2 - e^{-t/\tau_1} - e^{-10t/\tau_1})$$

The coefficients corresponding to a given creep curve can then be evaluated. It may be observed that  $\tau_M$ ,  $\alpha_1$ , and  $\tau_1$  are stress dependents and would thus vary with applied stress even for the same concrete. For low stress levels however, they may be considered constants for a given mix which renders the model linear. Freudenthal and Roll have evaluated these coefficients<sup>(105)</sup> for a wide range of mixes, both in their linear (i.e., stress-independent) and nonlinear (i.e., stress-dependent) ranges. Good correlation was obtained between predicted and actual behavior during a supplementary test. The model coefficients have also been checked against observed recovery data.<sup>(150)</sup> In general, better conformity to the viscoelastic model was observed for richer mixes than for leaner ones.

Under certain circumstances further simplification of the model is possible. A more detailed discussion of the simplified model is included elsewhere in the report under structural analysis.

#### F. MODEL DUE TO TORROJA AND PAEZ

The rheological model proposed by Torroja and Paez<sup>(66)</sup> is rather elaborate, hence its elementary counterpart (Figure 23-a) will be discussed first. This elementary model consists of four elements: i.e., an elastic spring H, a spring B enclosed in a casing and transferring load to the casing through friction, a similar spring D with friction working in parallel with a dashpot N. The last two elements are combined in parallel. The two together are connected in series with the other two.

The spring H is a simple linear spring and corresponds to an instantaneous reversible response typical of an ideal Hookean solid.

In the spring B with friction, if subjected to a tensile load by pulling the spring at one



end and the casing at the other, only as many spirals will move as will have their frictional resistance with the casing overcome -- the total frictional resistance coming into play being equal to the external load applied. When a load equal to the maximum friction that can be mobilized is applied, fracture may be said to have taken place. Each successive spiral towards the free end of spring will take a decreasing amount of load. If the load is below the fracture limit, one or more spirals at the loose end may have no load at all. If the load is removed, the spirals closer to the originally loaded end will tend to recede and recovery of deformation takes place. However, as the free end is approached, each successive spiral will contract less due to the constant friction that it has to overcome to return to its initial position, thus resulting in a certain amount of permanently irrecoverable deformation after removal of the load.

Under loads applied at high rates only the springs H and B may be assumed to undergo deformation. The stress-strain curve for such loading will then take the form of a second degree parabola, with a slope gradually decreasing at an increasing rate to zero at the vertex which corresponds to the ultimate stress.

If, before the load reaches the ultimate value the model is unloaded, a residual strain remains when the load reaches zero, its magnitude depending upon the stress level at unloading. If the load is increased again, the stress-strain curve will rise until the stress reaches the maximum value attained before the unloading, after which it will follow the curve it would have followed if it had not been unloaded. This general behavior corresponds fairly well to experimental observations.

The time-dependence of the deformations of the model is simulated by the elements N

and D. When a load is applied to the combination N D, a typical Kelvinian behavior will be observed, i.e., the element D will take a gradually increasing load, tending asymptotically to a value equal to the full load applied. The spring D, however, has a frictional component which makes it nonlinear with respect to the applied stress. Hence as it takes up more and more load, more and more spirals are stressed. Hence the spring progressively "softens." Under a constant applied load, the strain will gradually increase to an asymptotic value, but this value will not be proportional to the stress applied.

The observed reduction in strength of concrete under sustained loads is explained by assuming the spring D to be shorter than the spring H. Under a high rate of loading, the dashpot N will act as a nearly rigid body. Hence the spring D will take no load, the fracture being governed by the spring B. However, under sustained loads the spring D will take up load. Fracture will occur if at any stage the stress in spring B or D exceeds the respective ultimate resistance. The ultimate resistance of spring D will limit the load that can be sustained for infinite time without fracture taking place. At any sustained load of intermediate magnitude fracture will take place after different periods, depending upon the magnitude of the sustained load.

A more detailed form of the above model is shown in Figure 23-b. The general pattern is the same except that the elements N and D of the elementary model have been condensed into one element by assuming the casing of the spring D to be connected to a reservoir of a viscous liquid, thus eliminating the dashpot N. Several such elements have been combined in series to form chains, the elements

differing from each other in their characteristic values of the coefficients of viscosity, friction, and elasticity. Several such chains have been combined in parallel. The whole complex may now be considered as representing a concrete specimen as regards its rheological behavior.

Shrinkage in concrete can also be represented by this model. If the fluid level in the reservoir is reduced by desiccation, a suction is created in the casings which pulls the springs inwards, causing shrinkage. If neighboring parts of the system lose unequal quantities of the fluid the resulting differential tensile stresses may reach levels high enough to cause rupture of several chains, which may be identified with shrinkage cracking in concrete.

The model presented appears to simulate many aspects of the rheological behavior of concrete. However some of the elements of the model will probably have to be made dependent on time, load, or deformation for more complete representation. A free dashpot should be introduced in each chain to simulate the phenomenon of stress relaxation in concrete.

#### G. MODELS DUE TO CHANG AND KESLER

Two nonlinear models were proposed<sup>(70, 71)</sup> to represent the time-dependent behavior of concrete. Each model simulates only certain characteristics. The work is further discussed elsewhere in this review, under the possible correlation of sonic and creep properties of concrete.

The first model consists of a Kelvin body with a strain-softening spring and a time-thickening dashpot. The spring coefficient is assumed to be proportional to the reciprocal of the creep strain and the viscosity of the dashpot, as a power function of creep time. Thus the expressions for the spring and the dashpot coefficients are:

$$k_K = \frac{k_1}{\epsilon_c} \text{ and } c_K = c_1 t_c^a$$

where

$k_1$  = spring constant when  $\epsilon_c$  is unity

$\epsilon_c$  = creep strain

$c_1$  = dashpot coefficient when  $t_c$  is unity

$t_c$  = creep time

$a$  = constant.

The creep-time relation of such a model is given by the expression

$$\epsilon_c = \epsilon_{c1} t_c^{(1-a)}$$

where

$$\epsilon_{c1} = \left( \frac{\sigma - k_1}{c_1} \right) \left( \frac{1}{1-a} \right).$$

This expression represents creep as a power function of time which would be a straight line on a logarithmic plot. Creep data obtained by the authors corroborated this trend. From the observed creep-time data, the coefficients for the model were evaluated.

The second model is a Maxwell body with a stress-softening spring and a time-thickening dashpot. Thus the corresponding expressions for the spring and the dashpot coefficients are:

$$k_M = \frac{k_2}{\sigma} \\ c_M = c_2 t_c^b$$

respectively, where

$k_2$  = spring constant for a sustained stress equal to unity

$\sigma$  = sustained stress

$c_2$  = dashpot coefficient for creep time equal to unity

$t_c$  = creep time

$b$  = a constant.

The creep-time relation of such a model is given by the expression



$$\epsilon_c = \epsilon_{c2} t_c^{(1-b)}$$

where

$$\epsilon_{c2} = \frac{\sigma}{c_2(1-b)}.$$

This expression is also a power function of time. Coefficients corresponding to this model were evaluated as for the first one from logarithmic plots of experimental creep data. It may be observed that the first model can simulate creep and time-dependent recovery only. Instantaneous deformation under load and stress relaxation under sustained deformation are not represented. On the other hand the second model can simulate creep, recovery, and instantaneous elastic behavior, but does not represent creep recovery or delayed elasticity.

#### H. HANSEN'S MODEL

From an analysis of the nature of creep, and from considerations of the structure of concrete and cement paste, Hansen<sup>(78)</sup> devised a model for representing the rheological behavior of concrete. The model (Figure 51) consists of six elements -- three springs and three dashpots -- and can be represented by the structural formula

$$H_1 - [N_1 | H_2 - N_2 - (H_3 | N_3)]$$

where H is a spring (Hookean body), N, a dashpot (Newtonian body). The Burgers body  $H_2 - N_2 - (H_3 | N_3)$  represents the cement paste, the dashpot  $N_1$  represents the voids, and the spring  $H_1$ , the aggregate.

When simplified, the model becomes identical with the one suggested by Flügge<sup>(55)</sup> (Figure 48), described earlier.

#### I. GLUCKLICH'S MODELS FOR HARDENED CEMENT PASTE

Glucklich<sup>(151)</sup> proposed two alternative rheological models to explain the mechanical behavior of hardened cement paste.

The first model (Figure 52-a) consists of two Kelvin bodies in series (one with a nonreturn valve) which are in turn connected in series with a set of springs  $H_1$  -- themselves connected in series with varying amounts of frictional restraint between the individual springs. When a load is instantaneously applied, an instantaneous deformation will be observed. A part of this will be reversible, being due to the springs  $H_1$ , and a part of this will be irreversible, being due to slippage of the ends of those springs for which the limiting frictional forces were smaller in magnitude than the applied load. The instantaneous deformation will then be followed by the delayed deformation or creep of the two Kelvin bodies: the reversible deformation of the first Kelvin body being identified with the reversible or primary creep, the irreversible deformation of the second Kelvin body being identified with the irreversible or the secondary creep. Glucklich obtained a curve for the secondary creep from the total creep curve by subtracting from it the permanent set and the asymptotic values of observed recovery. He found that the secondary creep terminated after a relatively short time, suggesting that hardened paste was essentially a solid rather than a liquid. Physically, he interpreted the reversible delayed deformation as linked with flow of water within the saturated zones and the non-reversible creep as a reflection of the flow of water into undersaturated zones which is rendered non-reversible by virtue of capillary forces and the surface tension of water.

The second model (Figure 52-b) consists of a set of frictioned springs in series, like those in the first model, connected in series with a Kelvin body with a reversible dashpot. Its spring element is composed of another

series of friction connected springs. A sustained load on this model would result in an instantaneous deformation, partly reversible and partly irreversible. This will then be followed by a delayed deformation continuously increasing at a decreasing rate, a part of which is not recoverable on removal of the load. Here, however, the irrecoverable part will be not only a function of the applied load but also of the deformation. This irrecoverable deformation is identified with the macroscopic manifestation of the formation of minute cracks in the course of the deformation of the Kelvin body. The secondary creep,

according to this model, may be visualized as a delayed permanent set.

Glucklich and Ishai<sup>(111)</sup> also suggest another simplified model based on torsion tests conducted on cement mortar specimens at various degrees of desiccation. The model is shown in Figure 53 and broadly simulates the observed influence of the evaporable moisture content on deformation characteristics under low stresses. The hardened mortar is considered, as before, to consist of a continuous elastic phase and a discontinuous fluid phase. For a more complete representation, they suggest several such mechanisms in series.

## IX. DYNAMIC PROPERTIES OF CONCRETE

### A. GENERAL

Dynamic testing (as the term will be used here) consists essentially of exciting the object under study into a state of steady or transient oscillatory motion and measuring its relevant response. Material properties obtained from such measurements are referred to as dynamic properties. Dynamic tests may be classified as destructive or nondestructive, however, the scope of this report does not include destructive dynamic testing. Excellent discussions of available nondestructive test techniques and their evaluation are presented by Hinsley,<sup>(153)</sup> McGonnagle,<sup>(154)</sup> and Whitehurst and Parker.<sup>(197)</sup>

The term "dynamic testing" covers tests involving measurement of the frequencies and amplitudes of specimens under free or forced damped vibrations, tests in which the velocity and attenuation of elastic waves or pulses travelling through the specimen are measured, and sclerometric techniques in which the rebound of a hammer or the size of the indentation left by it on the specimen surface is measured. The term "sonic testing" is often applied to tests where the frequencies encountered lie in the audio-range, i.e., approximately between 20 to 20,000 cycles per second. It is sometimes restricted to cases where the specimen is subjected to sustained oscillations only. In the present review, however, the terms sonic and dynamic tests will be used

interchangeably unless specifically defined.

Dynamic testing techniques offer a wide scope of practical applications both in industry and research. Detection of surface and internal flaws, thickness gaging, measurement of elastic and damping characteristics of materials, evaluation of aging and durability of products, studies of relaxation and creep characteristics -- these are only a few of such applications. A comprehensive study of "sonics" and its applications to a wide variety of problems has been presented by Heuter and Bolt.<sup>(155)</sup> Kesler and Chang<sup>(156)</sup> have reviewed available sonic techniques for studying the mechanical properties of materials. The present discussion will be limited primarily to the latter aspect of dynamic testing, with particular emphasis on applications to concrete.

### B. DETERMINATION OF MECHANICAL PROPERTIES FROM DYNAMIC TESTS

The study of a body subjected to vibration provides information about two very important material characteristics, namely the elastic and the dissipative or damping.

Elastic properties can be studied by measuring the frequency of a body of known size, mass, and shape, under forced or free vibration or alternatively by measuring the velocity of elastic waves or pulses travelling through the material.

The dissipative or damping characteristics

can be studied either by measuring the steady-state amplitudes of the response at near-resonant frequencies under forced vibration or by measuring the decaying amplitude during free vibrations. The loss of energy may also be estimated from the observed rise of temperature due to dissipation of the vibrational energy as heat.

For purposes of discussion, the above techniques may be conveniently grouped as follows:

a. Principal Frequency Methods

Constant amplitude (forced vibration)

Decaying amplitude (free vibration)

b. Wave Transmission Methods

Pulse velocity

Standing wave

C. PRINCIPAL FREQUENCY METHODS

In the case of materials with relatively small dissipation of vibrational energy, the effect of damping on the principal frequencies of a specimen is negligible. The frequency of damped free vibration of the resonant frequency under forced vibration may thus be used as a close estimate of the undamped natural frequency of the system. For a given shape, size, and mass of the specimen, the elastic constants can then be expressed in terms of the natural frequencies corresponding to the relevant mode of vibration. The Young's modulus is thus related to the frequencies in longitudinal and transverse modes, while the shear modulus is related to the frequency in the torsional mode.

The following relations can be established between the dynamic values of the elastic constants and the corresponding natural frequencies:

$$E_d = Aw(f_n)^2 = Bw(f'_n)^2$$

$$G_d = Cw(f''_n)^2$$

$$\mu = \frac{E_d}{2G_d} - 1$$

where:

$E_d$  = dynamic value of the Young's modulus of elasticity

$G_d$  = dynamic value of the elastic modulus of elasticity

$\mu$  = Poisson's ratio

$f_n$  = natural frequency of flexural vibration

$f'_n$  = natural frequency of longitudinal vibrations

$f''_n$  = natural frequency of torsional vibrations

$w$  = weight of the specimen

A, B, C = factors depending on the shape and size of the specimen, and the mode of vibration.

The values for the factors A, B, and C for specimens in the shapes of rectangular prisms and cylinders with different ratios have been investigated by Pickett<sup>(157)</sup> and are incorporated in the ASTM Designation C215-60.

The values of the elastic moduli obtained from dynamic tests are not necessarily the same as those obtained from conventional static tests. In the case of materials which exhibit nonlinear stress-strain response, for example concrete, the dynamic moduli would correspond roughly to the initial tangent moduli obtained from static tests.

As mentioned earlier, the procedure for determining the dynamic moduli of elasticity is identical, once the natural undamped frequency has been estimated, either from a free vibration test or a resonance test. The determination of the damping characteristics, however, proceeds differently for the two methods.

In the case of the resonant test, the

procedure consists of determining the resonant frequency  $f_n$  and frequencies  $f_1$  and  $f_2$  on either side, for which the amplitude is a known fraction  $\frac{1}{n}$  of the corresponding amplitude at resonance as shown in Figure 54. For small damping, the logarithmic decrement  $\delta$  is then given by:

$$\delta = \frac{\pi}{\sqrt{n^2 - 1}} \frac{f_2 - f_1}{f_n}.$$

For convenience  $n$  is usually taken as  $\sqrt{2}$ , when

$$\delta = \frac{\pi(f_2 - f_1)}{f_n}.$$

For the decaying amplitude method (Figure 55) the parameter  $\delta$  is given by:

$$\delta = \ln \frac{A_n}{A_n + 1} = \frac{1}{m} \ln \frac{A_n}{A_n + m}$$

where  $A_m, A_n \dots$  are the amplitudes at the  $m^{\text{th}}, n^{\text{th}} \dots$  cycle, from any arbitrary origin.

#### 1. Equipment for the Constant Amplitude Test

A block diagram for a typical setup for the constant amplitude or resonant frequency test is shown in Figure 56. The driving unit is powered by the amplified output of a variable frequency oscillator and excites a suitably supported specimen. Piezoelectric, magnetostriction, and electrodynamic drivers have been successfully used. The response of the specimen is sensed by a pickup in contact with it, amplified and fed into a voltmeter or other voltage indicating device. An oscilloscope for checking the mode in which the specimen is vibrating, and a counter for precise determination of frequencies, constitute useful supplementary equipment.

The experimental procedure consists of setting up the specimen on suitable supports, exciting it into the required mode of vibration, and observing the pickup output as the oscillator frequency is varied. The frequency

corresponding to the maximum output of the pickup, which indicates the maximum amplitude, is recorded together with the pair of frequencies corresponding to outputs equal to  $1/\sqrt{2}$  times the resonance level. The dynamic moduli and logarithmic decrement can then be calculated as indicated earlier.

Details of suitable supports and methods of locating and mounting the driving and pickup units for longitudinal, flexural, and torsional vibrations are given in ASTM Designation C251. (158)

#### 2. Equipment for the Decaying Amplitude Test

In carrying out a decaying amplitude test, the specimen is supported as for the constant amplitude test, but the excitation is affected by a single impact which causes the specimen to vibrate freely in its natural frequency. As there is no further application of external force, the amplitude decays due to the damping present in the system. The output of the pickup is displayed on an oscilloscope. A single sweep, triggered by the initial impact, permits convenient observation or photographing of the decay pattern. A block diagram of the setup is shown in Figure 57.

The dynamic moduli of elasticity and the logarithmic decrement is calculated from the observed frequency and the decay curve of the amplitude, as described earlier.

#### D. WAVE TRANSMISSION METHODS

In contrast with the principal frequency methods where the frequency and amplitude are studied with the specimen vibrating in a principal mode, the wave transmission techniques are based on the manner in which a continuous wave or isolated sonic pulses are propagated through the material under test.

When a sonic wave passes through an elastic medium, three kinds of waves are

generated, i.e., compressional or longitudinal, shear or transverse, and surface or Rayleigh waves. Of these three types of waves the longitudinal waves, which travel fastest, are used for sonic transmission tests.

The dynamic modulus of elasticity  $E_d$  can then be expressed in terms of the observed velocity of longitudinal waves as follows:

$$E_d = \rho v^2 (1 + \mu) (1 - 2\mu) / (1 - \mu) = F \rho v^2$$

where:

$\rho$  = density of the material

$v$  = velocity of the longitudinal wave

$\mu$  = Poisson's ratio

$$F = \frac{(1 + \mu)(1 - 2\mu)}{(1 - \mu)}$$

The factor  $F$  is not very sensitive to changes in  $\mu$ , and may, for practical purposes, be assumed to be the same for materials having Poisson's ratios of a comparable magnitude.

The above relation between the dynamic modulus and the transmission velocity is also applicable to intermittent disturbances of constant frequencies or isolated pulses of mixed frequencies. Application to the latter is justifiable only if the transmission velocity is independent of the frequency. Most engineering materials appear to more or less satisfy this criterion.

A variety of techniques have been used for generating the continuous waves or isolated pulses in the material, and for measuring their velocity. Some of the commonly used methods are briefly described.

#### 1. Methods of Excitation

Piezoelectric crystal transducers driven by the amplified output from an oscillatory or pulse generator have been widely used<sup>(159-165)</sup> for generating the necessary elastic waves in the material. Considerably increased power outputs have been made possible by the recently developed synthetic crystals. Magnetostriction

transducers provide a very convenient alternative source of both sonic and ultrasonic excitation.

Excitation by direct impact from manually operated or spring actuated hammers have also been used by many investigators<sup>(166-170)</sup> for generation of pulses.

#### 2. Measurement of Velocity

The most widely used approach consists of directly timing the pulse between two stations spaced a known distance apart and located along the direction of transmission. Suitable acoustic pickups are attached at the two stations and the time lag between the corresponding signals received by them is measured.

Two basic chronometric techniques have been successfully used. The first consists of modulating an oscilloscope beam by the output from the two pickups. The sweep is calibrated and synchronized with the first signal, which appears as a stationary "pip." The distance of the second signal on the screen is proportional to the elapsed time and can be scaled from the known sweep velocity. The soniscope used by Leslie and Cheesman,<sup>(159)</sup> Werden,<sup>(171)</sup> Lavery and Palmrose,<sup>(163)</sup> Caw-kell,<sup>(164, 165)</sup> and others is based on this method.

Alternatively, the pickups may be used to trigger a gating circuit (using electro-mechanical relays, thyratron tubes, or other devices) which lets through a timing signal. A direct current signal can be fed either to a ballistic galvanometer or a capacitor and voltmeter circuit. The total charge is measured, which in turn is proportional to the interval being measured.<sup>(167)</sup> A timing signal of known frequency can be used with a counter circuit to indicate the number of timing waves passing through the gate, which is then a measure of the time interval.<sup>(172)</sup>



In addition to direct chronometric methods, other techniques have been employed to measure the pulse velocity. A pulse is allowed to travel through a branched path of which the specimen forms one arm and a material with known elastic constants forms the other. The pickup locations in one or both arms is adjusted until the transit times become equal as shown by a null indicator. Actual time measurements are not necessary. The relative path lengths in the two arms are then used as a basis for determining the pulse velocity in the material under test.<sup>(173)</sup>

An alternative approach described by Long and Kurtz<sup>(166)</sup> utilizes a phase measurement technique. Part of the exciter input and the output of a movable pickup are fed to the horizontal and vertical plates of an oscilloscope. The pickup is moved away from the exciter until the phase difference  $180^\circ$  between the signals is indicated by the oscilloscope. The distance corresponds to one half wave length. As the exciting frequency is known, the velocity can be easily determined.

Block diagrams indicating the basic arrangements of various transmission setups are shown in Figures 58 through 60.

#### E. MERITS AND LIMITATIONS

As may be observed from the above descriptions, the principal frequency techniques can be used only if the mass, shape, and size of the specimen are known and the principal frequencies can be conveniently calculated. This requirement limits their use to the laboratory. The transmission techniques, on the other hand, can be used in the laboratory or on the field.

The principal frequency methods can be used for estimating both the elastic constants and damping characteristics, while the transmission approach is mainly suited for determining the elastic properties. Although it is

possible to estimate damping properties by measuring the extent of attenuation suffered by a wave or a pulse during transmission, the technique is best applied to materials with a high degree of damping using ultrasonic excitation. In addition, the determination is based on the assumption of distortion free wave propagation, which may be difficult to justify in most cases. Davies<sup>(174)</sup> has given a brief account of these methods.

Although one would expect that identical values of the elastic constants would be obtained from both principal frequency and transmission tests, significant differences have nevertheless been reported by some investigators. Batchelder and Lewis,<sup>(175)</sup> for example, found that the dynamic modulus obtained from velocity measurements was always greater than that from resonant frequency tests.

#### F. DYNAMIC TESTS ON CONCRETE

Dynamic testing techniques have found an increasing application in the study of the mechanical properties of plain concrete. Most of the studies have been limited, until recently, to the determination of the elastic constants and their direct or indirect application to other experimental problems. Such applications have included the assessment of damage due to freezing and thawing, shrinkage and cracking, location of cracks and flaws, estimation of strength, and other mechanical and physical properties, measurement of pavement thicknesses, and general quality control.

More recently, attempts have been made\* to correlate both elastic and damping characteristics, determined from dynamic tests with strength, fatigue, creep, and relaxation behavior of concrete.

Some of the more significant work on the

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\* See references 70, 71, 176, 177, 123.



dynamic properties of concrete and associated studies will be briefly reviewed.

One of the earliest experiments on dynamic testing of concrete was made by Powers.<sup>(178)</sup> He estimated the modulus of elasticity by measuring the natural frequencies of concrete and mortar bars by striking the specimens longitudinally and matching the note emitted with orchestra bells or a sonometer. He realized that the values of the dynamic modulus obtained in this way were higher (according to him truer, too) than the corresponding static values due to the fact that the dynamic tests were not complicated by plastic flow.

Hornibrook,<sup>(179)</sup> in 1939, reported his studies of freezing and thawing of concrete by using the change in dynamic modulus of elasticity as a criterion of deterioration. His equipment was better for dynamic tests than any used until then. He observed that the resonant frequency of a specimen was only slightly affected by variation in temperature in the case of relatively moist concrete, but oven drying appeared to have a considerable influence.

Thomson<sup>(180)</sup> appears to have been the first to study the damping properties of concrete. He used the band width of the resonance curve as a measure of the damping capacity (i.e., the amount of work dissipated as heat by internal friction in a unit volume of the material during a cycle of vibration) and the specific damping capacity (i.e., the damping capacity per unit of total vibrational energy). He also studied the effect of freezing and thawing cycles on the change in dynamic modulus of elasticity and the damping capacity of concrete specimens. He observed the phenomenon of the "double hump" (i.e., the occasional presence of two peaks in the amplitude-frequency curve), and attributed it to the presence of cracks in the specimen. Willis and

Reus, in the discussion of his paper, reported results of their experiments on dynamic testing of concrete and pointed out that the phenomenon of double hump was not wholly due to cracks.

Obert<sup>(181)</sup> measured dynamic modulus of concrete oven-dried under stress and observed that the difference between the elastic modulus at zero stress and a stress level of 25 per cent was about 10 per cent. In 1941, Obert and Duvall,<sup>(182)</sup> made a study of the variation of the dynamic modulus of elasticity and damping capacity, with change in moisture content and the size of specimen. They used both longitudinal and transverse modes of free and forced vibrations. They made several suggestions for standardization of the procedure for dynamic testing of concrete. They also reported that the dynamic modulus of a saturated specimen was close to that of a dry one, but that on resaturation, an oven-dried specimen did not return exactly to the original value. The damping capacity was found to vary sevenfold from oven-dried to saturated specimens. They therefore emphasized the specification of moisture content of the specimen when reporting the damping capacity.

Long and Kurtz<sup>(166)</sup> were probably the first to employ the pulse velocity techniques for testing concrete. Using a seismograph, they measured the velocity of sound pulses for studying the effects of membrane curing, standard curing, and no curing on the durability of concrete subjected to freezing and thawing. They also compared the dynamic moduli obtained by the wave velocity and resonant frequency methods and found the former to be higher. Long, Kurtz, and Sandenaw<sup>(167)</sup> used an electronic interval timer for precise measurement of the pulse velocity, and found that there was a consistent relation between

the dynamic modulus and the flexural strength of concrete. They presented a method for determining thickness of concrete pavements. Their paper includes an extensive bibliography of sixty entries on sonic testing of concrete.

The formulas for computing the elastic constants from fundamental transverse longitudinal and torsional frequencies of concrete specimens incorporated in ASTM Designation C215-60<sup>(158)</sup> are based mostly on Pickett's work.<sup>(157)</sup> In the original paper, Pickett discussed the approximations involved in the equations for deriving the elastic moduli and presented graphs giving correction factors for the effect of variation in Poisson's ratios. He also discussed the phenomenon of the double hump, referred to earlier. In another paper<sup>(183)</sup> he suggested equations for use in dynamic testing of concrete pavements.

Jones<sup>(184-188)</sup> employed the nondestructive dynamic testing methods in the study of concrete for purposes such as correlation of longitudinal wave velocity with cube strength, increase in wave velocity with setting of concrete and mortar, effect of freezing and thawing cycles, thickness measurement of road slabs, assessment of the degree of compaction and quality control of precast units. He also studied<sup>(189)</sup> the effect of frequency on the observed values of the dynamic modulus and damping factor. No significant influence could be discovered except that due to non-uniform drying.

Leslie and Cheesman<sup>(159)</sup> used ultrasonic pulse velocity measurements for field and laboratory testing of concrete. They used, as mentioned earlier, the soniscope for detecting the presence and extent of cracks in concrete dams. They also obtained satisfactory correlation between the dynamic moduli obtained from pulse velocity and resonant frequency tests.

Dynamic testing has been carried out on a considerable scale in France by Chefdeville, Dawance, Blance, and others.<sup>(168-170)</sup> The method has been employed for purposes such as detection of nonhomogeneity and presence of cracks in highway slabs and dams, study of the variation of the elastic modulus, and compressive strength. They gave empirical relationships between crushing strength of cubes and the dynamic moduli of elasticity. They also studied the effects of variables on the dynamic modulus of elasticity. In addition, discussions of the theory of dynamic testing and details of the equipment and experimental techniques developed in the laboratories of Building and Public Works have been described.

Pogamy<sup>(190)</sup> used dynamic testing to determine strength and correlating dynamic properties with porosity. Depelsenair<sup>(191)</sup> studied mechanical properties and internal friction of concrete by dynamic methods. Ultrasonic test methods were used by Eisenmann and Steinkamp to determine the depth and width of cracks in concrete. Lochner and Keet<sup>(192)</sup> determined the dynamic modulus of concrete from the measurement of the fundamental transverse frequency of concrete specimens using a simple calibrated Helmholtz resonator, the resonance of which was sensed by means of a stethoscope. The specimens were excited by striking them with a hammer.

Tests were carried out by Simmons<sup>(193)</sup> to determine values of Poisson's ratio of concrete by static and dynamic methods and to relate them to each other. Guha, Rao, and Ram<sup>(194)</sup> investigated the effect of cement-water ratio on the wave velocity in concrete.

Wesche<sup>(161,162)</sup> employed ultrasonic methods for the measurement of the depths and widths of cracks in concrete and described techniques for the determination of the elastic

modulus, strength, Poisson's ratio, and for studying the influence of impure water, setting of cement and concrete, nonuniformity of structure, location and depths of cracks, and deterioration of structure. Using a soniscope, Werden<sup>(171)</sup> tested strength and deterioration of concrete in dams, employing ultrasonic frequencies. Kaplan<sup>(195)</sup> reviewed developments in nondestructive testing methods and considered the ultrasonic pulse technique as the best for dynamic testing of concrete. He also investigated<sup>(196)</sup> the effect of voids due to incomplete consolidation on the compressive and flexural strength of concrete, ultrasonic pulse velocity and the dynamic modulus of elasticity of concrete.

Whitehurst and Parker<sup>(197)</sup> have given a brief review of dynamic tests in concrete in the ASTM Bulletin number 169. In other papers<sup>(198, 199)</sup> Whitehurst reviewed pulse velocity techniques and equipment, described new applications in nondestructive testing, and discussed determination of sources of error. Armstrong, Ulp, and Larson<sup>(200)</sup> used dynamic tests for evaluating the moment of inertia of concrete block sections with one to three holes. They suggested the possibility of using dynamic tests for determining the moments of inertia for the study of stresses in honeycomb sections. Chaturvedi, Gupta, and Shrivastava<sup>(201)</sup> investigated the quality of concrete using a dynamic method, and indicated that the dynamic modulus can be correlated with most of the important mechanical properties of concrete.

Laverty and Palmrose<sup>(163)</sup> employed the soniscope with ultrasonic pulse propagation techniques for testing the performance of concrete in dams. Both the resonant frequency and ultrasonic pulse propagation methods were used by Cawkell<sup>(164, 165)</sup> for nondestructive testing of concrete. Transmission tests were

successfully used for testing the quality of concrete up to 20 feet thick.

Except for the work of Thomson,<sup>(180)</sup> Obert and Duvall,<sup>(182)</sup> and Jones,<sup>(189)</sup> hardly any of the earlier investigators have attempted to determine the damping characteristics of concrete by dynamic tests or to correlate them with other properties of concrete. A series of investigations have been recently carried out and further work is in progress at the University of Illinois, aimed towards correlating the elastic and damping characteristics obtained from dynamic tests with the strength, creep, and fatigue behavior of concrete. Kesler and Higuchi<sup>(176)</sup> and Chang<sup>(70, 71, 124)</sup> have investigated the relationship of sonic properties with compressive and flexural strengths, creep, and relaxation. Girgrah and Kesler<sup>(148)</sup> have recently reported a study of the damping properties of concrete as related to the creep behavior and the time-variation of the dynamic modulus and logarithmic decrement. A review of the findings of Chang and Kesler on the correlation of creep and relaxation with dynamic properties of concrete<sup>(70, 71)</sup> is included elsewhere in this report. The possibility of predicting creep in concrete from the knowledge of the dynamic modulus and the logarithmic decrement have been further investigated by Kesler,<sup>(202)</sup> Baker,<sup>(149)</sup> and Vaishnav.<sup>(123)</sup>

#### G. DAMPING OF VIBRATIONS

A real body undergoing vibration sooner or later comes to a stop if external energy is not supplied to it. Alternatively, if a state of sustained vibration has to be maintained, a continuous input of energy is necessary to compensate for the loss due to internal or external damping. The present discussion will be limited to internal damping in materials in general and concrete in particular.

The energy per unit volume of the material, dissipated internally per cycle of vibration, provides a measure of the damping present and is referred to as the damping capacity of the material. The damping capacity expressed as a fraction of the total energy of vibration is called the specific damping capacity of the material.

The logarithmic decrement  $\delta$  is a very convenient parameter for experimental determination of damping capacity, and is defined as follows:

$$\delta = \ln \frac{A - \Delta A}{A}$$

where:

$A$  = the amplitude during any cycle of free vibration

$\Delta A$  = the diminution of the amplitude during the complete cycle.

For small damping capacities, the following approximate relations are justifiable:

$$\delta = \frac{\Delta A}{A} = \frac{\psi}{2}$$

where:

$\psi$  = the specific damping capacity.

The above relationships apply at any instant to any vibrating system, in any mode and regardless of the type of damping. Of course,  $\delta$  may not necessarily be a constant in all cases.

Types of Damping:

In general, the type of damping present in a material consists of one or more of the following types:

1. Viscous Damping: This behavior is characterized by a dissipative force proportional to the velocity, and opposite in direction to it. The following differential equation governs the resultant motion.

$$m\ddot{x} + c\dot{x} + kx = 0$$

where:

$m$  = effective mass of the system

$c$  = coefficient of damping

$x, \dot{x}, \ddot{x}$  = the displacement and its first and second time derivative respectively

$k$  = effective elastic constant of the system.

For small damping the logarithmic decrement is then given by:

$$\delta = \frac{\pi c}{\sqrt{km}} = \frac{\pi c}{m\omega_n}$$

where  $\omega_n$  is the natural undamped angular frequency of the system.

As may be observed from the above relationship, the damping capacity in viscously damped systems is independent of the amplitude and is inversely proportional to the frequency. The amplitude decay curve is exponential.

2. Coulomb Friction Damping: This type of damping produces a constant dissipative force acting opposite in direction to the velocity. The resultant motion is given by:

$$m\ddot{x} = kx = \pm F$$

where  $F$  is the magnitude of the constant frictional force. The logarithmic decrement for small damping is then given by:

$$\delta = \frac{4F}{m\omega_n^2 A}$$

The specific damping capacity is thus seen to be a function of the amplitude  $A$  and the frequency  $\omega_n$ . The amplitude decay curve is linear.

3. Solid or Hysteresis Damping: The dissipative force associated with this type of damping is proportional to the displacement and is independent of the frequency.

If the dissipative force  $F = \lambda x$ , then for small damping and assumed harmonic motion, the logarithmic decrement  $\delta$  is given by:



$$\delta = \frac{\pi \lambda}{k}$$

The damping capacity is thus independent of the amplitude and frequency. As in the case of viscous damping, an exponential decay of amplitude is exhibited.

4. Turbulent Fluid Damping: The dissipative force is proportional to the square of the velocity and acts in the opposite direction. The logarithmic decrement for small damping is given by:

$$\delta = \frac{\pi \gamma A}{m}$$

where the dissipative force  $F = \gamma \dot{x}$ . The damping capacity is thus independent of the frequency but proportional to the amplitude.

Damping in which the forces are not in linear proportion to the velocity is referred to in general as nonlinear damping. The dependence or independence of the damping capacity on the amplitude and frequency is governed by the relation between the damping force and the displacement and its time derivatives. Rigorous analysis of nonlinearly damped systems is often very involved, and it is usual to replace the nonlinear damping force with an equivalent linear damping force so that the energy dissipated per cycle is the same in both cases.

#### H. DAMPING CHARACTERISTICS OF CONCRETE

Compared to the amount of work done on the study of damping in other materials, the efforts to investigate the nature and origin of damping in concrete have been infinitesimal. Most investigations of the dynamic properties of concrete have been limited, as indicated earlier, to studies of the elastic constants and the factors influencing them.

The lack of emphasis on the damping behavior of concrete has perhaps been due as much to the inherent experimental problems involved as to an insufficient realization of

the possibilities of using damping capacity as an index of important physical and mechanical properties. The absence of the direct utility of damping characteristics, as such, in conventional engineering design in concrete is another possible reason. It is only recently that efforts have begun to be directed in this direction. The very limited experimental information available on this aspect will be briefly reviewed. In addition, tentative suggestions regarding the probable nature and origin of damping, based on available knowledge of the structure of concrete, and possible trends of further research will be discussed.

Among the earlier investigators, Thomson<sup>(180)</sup> was perhaps the first to study damping in concrete and factors affecting it. Later, Obert<sup>(181)</sup> with Duvall<sup>(182)</sup> observed the relatively much higher sensitivity of damping to moisture content than that of the elastic properties. Little additional work was reported until the recent investigations undertaken at the University of Illinois by Kesler and associates during the last decade. Kesler and Higuchi<sup>(176)</sup> studied the effect of various organismic and environmental factors on the damping capacity of concrete. The logarithmic decrement was found to decrease with age and increase with moisture content. For relatively low moisture contents, the influence of the water cement ratio appeared to be small. An increase in frequency resulted in a slight increase in the logarithmic decrement.

Kesler and Chang<sup>(203)</sup> investigated the influence of frequency on the damping and elastic properties. The logarithmic decrement was found to be relatively insensitive to variations in frequency. Later observations by Jones<sup>(189)</sup> supported the findings.

Girgrah and Kesler<sup>(148)</sup> used the decaying amplitude technique to study the type of damping present in concrete and hardened neat

cement paste. They concluded that the damping is predominantly viscous, with a certain amount of Coulomb or frictional damping at low amplitudes.

More recently Baker and Kesler<sup>(204)</sup> investigated the frequency dependence of the logarithmic decrement and the form of the decay curve for mortars at various degrees of hydration and drying content. The tests indicated that both viscous and solid or hysteresis damping was present. Partially hydrated mortar appeared to be primarily viscously damped at lower frequencies and solidly damped at higher frequencies, while at later stages of hydration solid damping seemed to predominate throughout the frequency range of about 1000 to 4000 cycles per second, used for the tests. There appeared to be no evidence of any Coulomb or frictional damping.

Little specific information is available about the origin of damping in concrete and the associated mechanism. On the basis of the limited experimental findings discussed above and our current knowledge about the structure of hardened concrete, however, the following mechanisms have been tentatively listed<sup>(123)</sup> in which damping behavior could possibly originate.

1. Thermal Diffusion. When a stress is suddenly applied to a body, it undergoes strain in an adiabatic manner. If, however, the stress be applied very slowly, the resulting strain is obtained in isothermal manner. Any difference in the elastic moduli corresponding to the two conditions would manifest itself as delayed elasticity under sustained stresses and damping under cyclic stresses.

2. Viscous Friction. Knowledge of the structure of cement paste suggests that this mechanism might be quite significant in contributing to the damping in concrete. It is imaginable that the water adsorbed on the gel

particles may result in a significant dissipation of vibratory energy as a result of molecular diffusion of which viscous behavior is a manifestation. This dissipation would be temperature-sensitive and frequency-dependent.

3. Hysteresis or solid damping. Hysteresis has its origin in the solid phase of matter, primarily in crystal imperfections. The gel particles, if crystalline in structure, may exhibit hysteresis effects when subjected to vibratory disturbance. The aggregate particles too may possibly show a similar effect.

4. Intercrystalline slip. This mechanism may resemble the viscous friction at the grain boundaries in metals, and may be expected to participate in energy dissipation in concrete if the gel particles have a crystalline structure. Adjoining grain boundaries may have some disordered solid material in between as in case of metals, which may result in dissipation of energy in a viscous manner.

5. Intracrystalline slip. This phenomenon is characterized by the sliding of a crystal segment under applied stress along a plane inclined to the crystal axis, and stabilizing in the new position. Such a phenomenon could possibly occur both in the gel and aggregate in concrete and contribute to dissipation of vibrating energy.

6. Inertia losses in the capillary water. As the capillary water is held in position due to confinement only, the lag between its motion and that of the rest of the concrete could result in some dissipation of energy during vibration, especially if the capillary pores are partially full.

7. Moisture diffusion. A vibrating body of concrete or a body of concrete through which a wave is passing has different parts under different stress conditions. Under this

condition the moisture in the highly stressed part may tend to diffuse towards the lightly stressed adjoining part, thus dissipating energy.

8. Chemical diffusion. The gel water contains various salts in solution. Under different states of stress, the solubilities can vary and the dissolved salts may tend to diffuse from a region of lower solubility to an adjoining one of higher solubility, resulting in energy losses.

9. Air diffusion. The stress differential may cause entrapped and entrained air in the paste and pores of aggregates to diffuse from a highly stressed volume element to an adjoining lightly stressed one. This effect may not be significant in view of the fact that the volume changes and hence the pressure changes because the differential stress situation is likely to be of a low order of magnitude.

10. Rupture of bonds. Breaking of the adhesive bonds between the gel particles and aggregate particles or within the gel itself may also be responsible for some dissipation of energy. When groups of neighboring particles

suffer rupture of bonds the potential energy of vibration may be partly dissipated as surface energy of the resulting cracks.

11. Scattering. When the wave length of a transmitted sound wave is comparable to the size of gel particles or aggregate particles, this kind of loss may be expected to take place.

12. Particle rearrangement. Some of the gel particles may be under a state of unstable equilibrium. When subjected to a disturbance, they may tend to settle in more stable positions. This irreversible movement can result in some loss of energy during vibration.

As stated earlier, the above possibilities have been listed merely as plausible mechanisms of internal damping in concrete. The available state of knowledge is still inadequate to provide a basis for assigning relative magnitude to the contribution of the various factors or even lend definite support to the existence of some of them. A wide scope for research is open in this almost unexplored area, which might contain valuable clues and provide useful indices of the rheological and strength behavior of concrete.



## X. INTERRELATIONSHIP OF DYNAMIC AND CREEP PROPERTIES

### A. GENERAL CONSIDERATIONS

As has been discussed earlier, the phenomena of creep and damping are both manifestations of departures from perfect elasticity. In addition, almost all the organismic and environmental factors which influence one are also known to affect the other. Most hypotheses associate the same internal structural components and mechanisms with both phenomena.

In view of the above, the problem of seeking a correlation between the rheological behavior of a material under sustained stress and under cyclic stresses appears rational and worth studying. The possibilities of such a relationship have, in fact, been discussed by several investigators.

Leaderman<sup>(146)</sup> mentions that in 1835 Weber was perhaps the first to suggest the connection between the damping capacity of a material and the primary creep exhibited by it. Alexandrov and Lazurkin<sup>(205)</sup> and Alfrey<sup>(121)</sup> have further discussed the problem in connection with the response of high polymers, while Zener<sup>(143)</sup> studied the relationship between the damping capacity and anelasticity of metals.

In the case of viscoelastic systems, Alfrey<sup>(206)</sup> has pointed out that the associated rheological properties can be specified completely in terms of the relevant creep function, relaxation function, dynamic modulus function or any of four other alternative expressions.

The general analytical treatment involved in correlating these functions, however, become extremely complex for material represented by models more elaborate than a simple Kelvin body.

### B. PROBLEMS OF DYNAMIC ANALYSIS

The dynamic analysis of models involving more than one degree of freedom entails several difficulties. The major problem is the distribution of the inertial effects between the various components of the equivalent model. Even if some reasonable basis for such a distribution can be found, the analysis itself, even for the dynamic response of a simple four parameter model, is by no means easy. Scott and Finley<sup>(207)</sup> suggest an approach for the vibration analysis of a Burghers model.

Under certain conditions the analysis can be carried out without consideration of the inertial distributions. Within certain frequency ranges, it is often possible to simplify a multielement model to one with a smaller number of elements. For example, at not-too-low frequencies, a simple Burghers model may be treated, so far as the vibration response is concerned, as a three element model with the Maxwell dashpot eliminated. This is justifiable if the relaxation time of the Maxwell body is much greater compared to the time scale involved in the cyclic stresses applied. The free dashpot would thus "freeze" at all but extremely low frequencies. The resulting three

element model has been analyzed by Zener<sup>(143)</sup> and Alfrey<sup>(121)</sup> in slightly different forms. The behavior of such a simplified model (consisting of a Hookean body in series with a Kelvin body), when subjected to a sinusoidal external force with a variable frequency, is shown in Figure 61. The apparent dynamic modulus and the phase angle between the displacement and force are plotted against the forcing frequency.

#### C. APPLICATIONS TO CONCRETE

As discussed elsewhere in the review, concrete exhibits strength and deformation behavior characteristic of a viscoelastic material. Most investigators have suggested rheological models which consist of — or may be simplified to — a Burghers body.

For purposes of dynamic analysis the Burghers model may, as pointed out earlier, be simplified to a three element model within certain frequency ranges. Obviously under such an assumption the vibrational response would not involve the Maxwell dashpot, which is associated with the nonrecoverable deformation under sustained loads. Apparently, therefore, all the rheological elements may not necessarily be involved in both the behaviors. This raises a question as to the rationale of attempting to correlate the creep and dynamic behaviors of concrete. It does not, however, appear unreasonable to assume a covariance among the various elements, as the internal structure as a whole is a result of the same organismic and environmental factors. Such a covariance can then constitute a basis for the type of correlation sought.

#### D. EXPERIMENTAL EVIDENCE

Recent experimental investigations conducted at the University of Illinois have shown considerable promise. Several empirical correlations have been obtained between the creep and sonic properties of concrete for a limited

range of organismic and environmental factors. Chang and Kesler<sup>(70, 71)</sup> proposed power expressions for creep and relaxation functions of the form:

$$\epsilon_c = c_1 t_c^{c_2}, \text{ and } \sigma_r = \sigma_0 + c_3 t_r^{c_4}$$

where:

$\epsilon_c$  = creep strain after time  $t_c$

$\sigma_r$  = stress after time  $t_r$

$c_1, c_2$ , = coefficients depending on the stress level and concrete properties

$\sigma$  = sustained stress during creep

$\sigma_0$  = initial stress during relaxation.

The coefficients  $c_1, c_2, c_3$ , and  $c_4$  were then expressed in terms of the stress level ( $\sigma/f'_c$ ), dynamic modulus  $E_d$ , and the logarithmic decrement ( $\delta$ ) through the following functional relationships:

$$c_1 = f_1 (\sigma/f'_c) g_1 (E_d/f'_c) h_1 (\delta) \quad (5)$$

$$c_2 = f_2 (\sigma/f'_c) g_2 (E_d/f'_c) h_2 (\delta) \quad (5)$$

$$c_3 = f_3 (\sigma/f'_c) g_3 (E_d/f'_c) h_3 (\delta) \quad (5)$$

$$c_4 = f_4 (\sigma/f'_c) g_4 (E_d/f'_c) h_4 (\delta) \quad (5).$$

The functions  $f_1, 2, 3, 4, g_1, 2, 3, 4$  were then obtained by a process of iteration using experimental results from creep, relaxation, and sonic tests.

In later studies Vaishnav and Kesler<sup>(123)</sup> obtained relatively simple correlations between the logarithmic decrement of the saturated concrete at the age of 7 days with the unit creep strain corresponding to various combinations of loading age and creep time. In order to eliminate any nonlinearity at stress levels higher than about 30 per cent, a normalizing function  $N$  was evaluated and normalized values of the unit creep strains were used to simplify the expressions. The relationship had the general form:

$$N_c = \alpha \delta_{sat}^{-\beta}$$

where:

$N_c$  = normalized unit creep strain corresponding to a given age at loading and duration of creep

$\delta_{sat}$  = logarithmic decrement of saturated concrete at 7 days age

$\alpha, \beta$  = constants determined empirically from

creep and sonic data.

The logarithmic decrement at a particular moisture condition and degree of hydration was chosen arbitrarily, due to its better reproducibility. Similar correlations could be obtained with the damping properties at any other stage, if desired.

## XI. CONCLUDING REMARKS

### A. PRESENT STATE OF KNOWLEDGE

As may be seen from the foregoing review, the problem of creep in concrete has been studied from a variety of approaches. A majority of investigators have concentrated their efforts on the phenomenological rather than what might be termed the ontological aspects of creep. This is quite understandable in view of the immediate practical utility of the former. As a result, a mass of experimental data covering a wide range of the factors known to influence creep behavior has been accumulated over the past three or four decades.

A purely empirical approach, however, has its own limitations and this has been fully recognized in connection with the problem of creep. A number of attempts have therefore been able to explain the phenomenon in terms of the known or assumed basic structure of the material, and to present an integrated theory of creep. The inherent complexity of the problem and the multiplicity of variables involved has, unfortunately, hindered the development of a generally accepted concept. Although several very plausible hypotheses have been presented, none appear to be capable of explaining all the aspects of the phenomenon. In fact some hypotheses involve certain contradictions with observed behavior.

### B. UNSETTLED ISSUES

Despite the wealth of available experimental data, considerable differences of opinion

exist in regard to the quantitative — and in certain cases even qualitative — aspects of the phenomenon. Some of the points which await clarification and issues which are still unsettled are briefly discussed.

1. Isolation of Creep. Creep is arbitrarily defined as the difference between the total deformation under load and the sum of the instantaneous elastic and shrinkage deformations. The definition implies the independence of these deformations. Little information is available on this point, which has been frequently questioned and extensively discussed. Although of apparently little significance in an empirical approach, the question of mutual interaction of the deformations could assume considerable importance when a specific mechanism is to be associated with each type of deformation.

2. Magnitude of Creep. Although the qualitative effects of various factors on the magnitude of creep are well understood, quantitative estimation of the magnitude of creep for concrete of a given composition, situated in a given environment and subjected to a known stress history is, at best, an educated guess and a matter of judgment, unless an actual creep test can be performed under comparable conditions.

Another point that is still debatable is the existence or non-existence of a limiting magnitude of creep. Available experimental evidence is inadequate for arriving at an

unequivocal conclusion.

Most tests have been limited to uniaxial states of stress. Very limited information is available on creep under shear and multiaxial states of stress and the interrelationships, if any. The existence or non-existence of the counterpart of Poisson's ratio in creep is a controversial issue. Opinion is divided on the interpretation of experimental observation on this aspect.

3. Nature and Origin of Creep. The nature of the mechanism or mechanisms responsible for the phenomenon of creep in concrete is a point on which opinion is most sharply divided. This is not very surprising in view of the inherent difficulties of isolating and studying these mechanisms, which presumably originate at the microscopic or perhaps the molecular levels of the internal structure of the material.

Associated indirectly with the issue of the origin of creep is the problem of representing the behavior in the form of mathematical functions or analogous mechanical models in order to facilitate analysis and practical applications. The same controversies which exist on the origin of creep are reflected on this issue too.

#### C. PROMISING AREAS FOR FURTHER WORK

The innumerable and apparently unpredictable variables involved might appear to discourage any attempts directed towards the quantitative interpretation of creep in terms of the physico-chemical characteristics of concrete. The results of such studies, however, even if not wholly successful in their original aim, can be of great help in broadening our understanding of the problem, and can form

a rational basis for empirical treatments of more immediate practical value.

A deeper insight of the rheological properties and their origin can lead to the development of an alternative approach to the problem of routine estimation of the magnitude of creep, wherein other more conveniently measured characteristics could serve as indices for the former, whose direct measurement can be very involved and time consuming. One such characteristic is the damping behavior of hardened concrete.

The practical applications of the results of creep tests, performed under standardized conditions, to actual structural members is another problem which has engaged the attention of investigators. Although many possible procedures have been proposed and theoretical bases developed for them, most procedures are either too cumbersome to use for routine analysis or too simplified and seriously limited in scope. There appears to be great need for a practicable technique with general applicability, for the quantitative analysis of the influence of creep and relaxation on structural behavior.

Perhaps an obvious area, which has not been explored to the extent it deserves is the systematic observation of the time-dependent deformations in actual structures of selected types, situated in various climatic regions. Although a number of isolated observations are available, a concerted and organized effort in this direction appears very desirable. Any procedures developed on the basis of standardized tests and theoretical concepts can be judged only on the basis of the extent to which they can predict observed behavior in real situations.

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